Algebraic Reasoning (AR)

Kindergarten

MA.K.AR.1 Represent and solve addition problems with sums between 0 and 10 and subtraction problems using related facts.

MA.K.AR.1.1
For any number from 1 to 9, find the number that makes 10 when added to the given number.

MA.K.AR.1.2
Given a number from 0 to 10, find the different ways it can be represented as the sum of two numbers.

MA.K.AR.1.3
Solve addition and subtraction real-world problems using objects, drawings or equations to represent the problem.

Grade 1

MA.1.AR.1 Solve addition problems with sums between 0 and 20 and subtraction problems using related facts.

MA.1.AR.1.1
Apply properties of addition to find a sum of three or more whole numbers.

MA.1.AR.1.2
Solve addition and subtraction real-world problems using objects, drawings or equations to represent the problem.

Grade 2

MA.2.AR.1 Solve addition problems with sums between 0 and 100 and related subtraction problems.

MA.2.AR.1.1
Solve one- and two-step addition and subtraction real-world problems.

Grade 3

MA.3.AR.1 Solve multiplication and division problems.

MA.3.AR.2.1
Apply the distributive property to multiply a one-digit number and two-digit number. Apply properties of multiplication to find a product of one-digit whole numbers.

MA.3.AR.1.2
Solve one- and two-step real-world problems involving any of four operations with whole numbers.

Grade 4

MA.4.AR.1 Represent and solve problems involving the four operations with whole numbers and fractions.

MA.4.AR.1.1
Solve real-world problems involving multiplication and division of whole numbers including problems in which remainders must be interpreted within the context.

MA.4.AR.1.2
Solve real-world problems involving addition and subtraction of fractions with like denominators, including mixed numbers and fractions greater than one.

MA.4.AR.1.3
Solve real-world problems involving multiplication of a fraction by a whole number or a whole number by a fraction.

Grade 5

MA.5.AR.1 Solve problems involving the four operations with whole numbers and fractions.

MA.5.AR.1.1
Solve multi-step real-world problems involving any combination of the four operations with whole numbers, including problems in which remainders must be interpreted within the context.

MA.5.AR.1.2
Solve real-world problems involving the addition, subtraction or multiplication of fractions, including mixed numbers and fractions greater than 1.

MA.5.AR.1.3
Solve real-world problems involving division of a unit fraction by a whole number and a whole number by a unit fraction.

Grade 6

MA.6.AR.1 Apply previous understanding of arithmetic expressions to algebraic expressions.

MA.6.AR.1.1
Given a mathematical or real-world context, translate written descriptions into algebraic expressions and translate algebraic expressions into written descriptions.

MA.6.AR.1.2
Translate a real-world written description into an algebraic inequality in the form of $x > a$, $x < a$, $x \leq a$ or $x \geq a$. Represent the inequality on a number line.

MA.6.AR.1.3
Evaluate algebraic expressions using substitution and order of operations.

MA.6.AR.1.4
Apply the properties of operations to generate equivalent algebraic expressions with integer coefficients.

Grade 7

MA.7.AR.1 Rewrite algebraic expressions in equivalent forms.

MA.7.AR.1.1
Apply properties of operations to add and subtract linear expressions with rational coefficients.

MA.7.AR.1.2
Determine whether two linear expressions are equivalent.

Grade 8

MA.8.AR.1 Generate equivalent algebraic expressions.

MA.8.AR.1.1
Apply the Laws of Exponents to generate equivalent algebraic expressions, limited to integer exponents and monomial bases.

MA.8.AR.1.2
Apply properties of operations to multiply two linear expressions with rational coefficients.

MA.8.AR.1.3
Rewrite the sum of two algebraic expressions having a common monomial factor as a common factor multiplied by the sum of two algebraic expressions.

Grades 9-12

MA.912.AR.1 Interpret and rewrite algebraic expressions and equations in equivalent forms.

MA.912.AR.1.1
Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.

MA.912.AR.1.2
Rearrange equations or formulas to isolate a quantity of interest.

MA.912.AR.1.3
Add, subtract and multiply polynomial expressions with rational number coefficients.

MA.912.AR.1.4
Divide a polynomial expression by a monomial expression with rational number coefficients.

MA.912.AR.1.5
Divide polynomial expressions using long division, synthetic division and algebraic manipulation.

MA.912.AR.1.6
Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of polynomials.

MA.912.AR.1.7
Rewrite a polynomial expression as a product of polynomials over the real number system.

MA.912.AR.1.8
Rewrite a polynomial expression as a product of polynomials over the real or complex number system.

MA.912.AR.1.9
Apply previous understanding of rational number operations to add, subtract, multiply and divide rational algebraic expressions.

MA.912.AR.1.10
Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of rational algebraic expressions.

MA.912.AR.1.11
Apply the Binomial Theorem to create equivalent polynomial expressions.

MA.K.AR.2 Develop an understanding of the equal sign.	MA.K.AR.2.1 Explain why addition or subtraction equations are true using objects or drawings.	**MA.1.AR.2** Develop an understanding of the relationship between addition and subtraction.	MA.1.AR.2.1 Restate a subtraction problem as a missing addend problem using the relationship between addition and subtraction. MA.1.AR.2.2 Determine and explain if equations involving addition or subtraction are true or false. MA.1.AR.2.3 Determine the unknown whole number in an addition or subtraction equation, relating three whole numbers, with the unknown in any position.	**MA.2.AR.2** Demonstrate an understanding of equality and addition and subtraction.	MA.2.AR.2.1 Determine and explain whether equations involving addition and subtraction are true or false. MA.2.AR.2.2 Determine the unknown whole number in an addition or subtraction equation, relating three or four whole numbers, with the unknown in any position.	**MA.3.AR.2** Develop an understanding of equality and multiplication and division.	MA.3.AR.2.1 Restate a division problem as a missing factor problem using the relationship between multiplication and division. MA.3.AR.2.2 Determine and explain whether an equation involving multiplication or division is true or false. MA.3.AR.2.3 Determine the unknown whole number in a multiplication or division equation, relating three whole numbers, with the unknown in any position.	**MA.4.AR.2** Demonstrate an understanding of equality and operations with whole numbers.	MA.4.AR.2.1 Determine and explain whether an equation involving any of the four operations with whole numbers is true or false. MA.4.AR.2.2 Given a mathematical or real-world context, write an equation involving multiplication or division to determine the unknown whole number with the unknown in any position.	**MA.5.AR.2** Demonstrate an understanding of equality, the order of operations and equivalent numerical expressions.	MA.5.AR.2.1 Translate written real-world and mathematical descriptions into numerical expressions and numerical expressions into written mathematical descriptions. MA.5.AR.2.2 Evaluate multi-step numerical expressions using order of operations. MA.5.AR.2.3 Determine and explain whether an equation involving any of the four operations is true or false. MA.5.AR.2.4 Given a mathematical or real-world context, write an equation involving any of the four operations to determine the unknown whole number with the unknown in any position.	**MA.6.AR.2** Develop an understanding for solving equations and inequalities. Write and solve one-step equations in one variable.	MA.6.AR.2.1 Given an equation or inequality and a specified set of integer values, determine which values make the equation or inequality true or false. MA.6.AR.2.2 Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers. MA.6.AR.2.3 Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers. MA.6.AR.2.4 Determine the unknown decimal or fraction in an equation involving any of the four operations, relating three numbers, with the unknown in any position.	**MA.7.AR.2** Write and solve equations and inequalities in one variable.	MA.7.AR.2.1 Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically. MA.7.AR.2.2 Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.	**MA.8.AR.2** Solve multi-step one-variable equations and inequalities.	MA.8.AR.2.1 Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides. MA.8.AR.2.2 Solve two-step linear inequalities in one variable and represent solutions algebraically and graphically. MA.8.AR.2.3 Given an equation in the form of $x^2 = p$ and $x^3 = q$, where p is a whole number and q is an integer, determine the real solutions.	**MA.912.AR.2** Write, solve and graph linear equations, functions and inequalities in one and two variables.	MA.912.AR.2.1 Given a real-world context, write and solve one-variable multi-step linear equations. MA.912.AR.2.2 Write a linear two-variable equation to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context. MA.912.AR.2.3 Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point. MA.912.AR.2.4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features. MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine domain constraints in terms of the context. MA.912.AR.2.6 Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically. MA.912.AR.2.7 Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context. MA.912.AR.2.8 Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.

	MA.2.AR.3 Develop an understanding of multiplication.	MA.2.AR.3.1 Represent an even number using two equal groups or two equal addends. Represent an odd number using two equal groups with one left over or two equal addends plus 1. MA.2.AR.3.2 Use repeated addition to find the total number of objects in a collection of equal groups. Represent the total number of objects using rectangular arrays and equations.	**MA.3.AR.3** Identify numerical patterns, including multiplicative patterns.	MA.3.AR.3.1 Determine and explain whether a whole number from 1 to 1,000 is even or odd. MA.3.AR.3.2 Determine whether a whole number from 1 to 144 is a multiple of a given one-digit number. MA.3.AR.3.3 Identify, create and extend numerical patterns.	**MA.4.AR.3** Recognize numerical patterns, including patterns that follow a given rule.	MA.4.AR.3.1 Determine factor pairs for a whole number from 0 to 144. Determine whether a whole number from 0 to 144 is prime, composite or neither. MA.4.AR.3.2 Generate, describe and extend a numerical pattern that follows a given rule.	**MA.5.AR.3** Analyze patterns and relationships between inputs and outputs.	MA.5.AR.3.1 Given a numerical pattern, identify and write a rule that can describe the pattern as an expression. MA.5.AR.3.2 Given a rule for a numerical pattern, use a two-column table to record the inputs and outputs.	**MA.6.AR.3** Understand ratio and unit rate concepts and use them to solve problems.	MA.6.AR.3.1 Given a real-world context, write and interpret ratios to show the relative sizes of two quantities using appropriate notations: a/b, a to b, or a:b where b ≠ 0. MA.6.AR.3.2 Given a real-world context, determine a rate for a ratio of quantities with different units. Calculate and interpret the corresponding unit rate. MA.6.AR.3.3 Extend previous understanding of fractions and numerical patterns to generate or complete a two- or three-column table to display equivalent part-to-part ratios and part-to-part-to-whole ratios. MA.6.AR.3.4 Apply ratio relationships to solve mathematical and real-world problems involving percentages using the relationship between two quantities. MA.6.AR.3.5 Solve mathematical and real-world problems involving ratios, rates and unit rates, including comparisons, mixtures, ratios of lengths and conversions within the same measurement system.	**MA.7.AR.3** Use percentages and proportional reasoning to solve problems.	MA.7.AR.3.1 Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems. MA.7.AR.3.2 Apply previous understanding of ratios to solve real-world problems involving proportions. MA.7.AR.3.3 Solve mathematical and real-world problems involving the conversion of units across different measurement systems.	**MA.8.AR.3** Extend understanding of proportional relationships to two-variable linear equations.	MA.8.AR.3.1 Determine if a linear relationship is also a proportional relationship. MA.8.AR.3.2 Given a table, graph or written description of a linear relationship, determine the slope. MA.8.AR.3.3 Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form. MA.8.AR.3.4 Given a mathematical or real-world context, graph a two-variable linear equation from a written description, a table or an equation in slope-intercept form. MA.8.AR.3.5 Given a real-world context, determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, a table, a graph or an equation in slope-intercept form.	**MA.912.AR.3** Write, solve and graph quadratic equations, functions and inequalities in one and two variables.	MA.912.AR.3.1 Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system. MA.912.AR.3.2 Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real and complex number systems. MA.912.AR.3.3 Given a mathematical or real-world context, write and solve one-variable quadratic inequalities over the real number system. Represent solutions algebraically or graphically. MA.912.AR.3.4 Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context. MA.912.AR.3.5 Given the x-intercepts and another point on the graph of a quadratic function, write the equation for the function. MA.912.AR.3.6 Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context. MA.912.AR.3.7 Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features. MA.912.AR.3.8 Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context. MA.912.AR.3.9 Given a mathematical or real-world context, write two-variable quadratic inequalities to represent relationships between quantities from a graph or a written description. MA.912.AR.3.10 Given a mathematical or real-world context, graph the solution set to a two-variable quadratic inequality.

					MA.7.AR.4 Analyze and represent two-variable proportional relationships.	MA.7.AR.4.1 Determine whether two quantities have a proportional relationship by examining a table, graph or written description. MA.7.AR.4.2 Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship. MA.7.AR.4.3 Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description. MA.7.AR.4.4 Given any representation of a proportional relationship, translate the representation to a written description, table or equation. MA.7.AR.4.5 Solve real-world problems involving proportional relationships.	**MA.8.AR.4** Develop an understanding of two-variable systems of equations.	MA.8.AR.4.1 Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations. MA.8.AR.4.2 Given a system of two linear equations represented graphically on the same coordinate plane, determine whether there is one solution, no solution or infinitely many solutions. MA.8.AR.4.3 Given a mathematical or real-world context, solve systems of two linear equations by graphing.	**MA.912.AR.4** Write, solve and graph absolute value equations, functions and inequalities in one and two variables.	MA.912.AR.4.1 Given a mathematical or real-world context, write and solve one-variable absolute value equations. MA.912.AR.4.2 Given a mathematical or real-world context, write and solve one-variable absolute value inequalities. Represent solutions algebraically or graphically. MA.912.AR.4.3 Given a table, equation or written description of an absolute value function, graph that function and determine its key features. MA.912.AR.4.4 Solve and graph mathematical and real-world problems that are modeled with absolute value functions. Interpret key features and determine domain constraints in terms of the context.

	MA.912.AR.5 Write, solve and graph exponential and logarithmic equations and functions in one and two variables.	MA.912.AR.5.1 Solve one-variable exponential equations using the properties of exponents. MA.912.AR.5.2 Solve one-variable equations involving logarithms or exponential expressions. Interpret solutions as viable in terms of the context and identify any extraneous solutions. MA.912.AR.5.3 Given a mathematical or real-world context, classify an exponential function as representing growth or decay. MA.912.AR.5.4 Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context. MA.912.AR.5.5 Given an expression or equation representing an exponential function, reveal the constant percent rate of change per unit interval using the properties of exponents. Interpret the constant percent rate of change in terms of a real-world context. MA.912.AR.5.6 Given a table, equation or written description of an exponential function, graph that function and determine its key features. MA.912.AR.5.7 Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context. MA.912.AR.5.8 Given a table, equation or written description of a logarithmic function, graph that function and determine its key features. MA.912.AR.5.9 Solve and graph mathematical and real-world problems that are modeled with logarithmic functions. Interpret key features and determine constraints in terms of the context.
	MA.912.AR.6 Solve and graph polynomial equations and functions in one and two variables.	MA.912.AR.6.1 Given a mathematical or real-world context, when suitable factorization is possible, solve one-variable polynomial equations of degree 3 or higher over the real and complex number systems. MA.912.AR.6.2 Explain and apply the Remainder Theorem to solve mathematical and real-world problems. MA.912.AR.6.3 Explain and apply theorems for polynomials to solve mathematical and real-world problems. MA.912.AR.6.4 Given a table, equation or written description of a polynomial function of degree 3 or higher, graph that function and determine its key features. MA.912.AR.6.5 Sketch a rough graph of a polynomial function of degree 3 or higher using zeros, multiplicity and knowledge of end behavior. MA.912.AR.6.6 Solve and graph mathematical and real-world problems that are modeled with polynomial functions of degree 3 or higher. Interpret key features and determine constraints in terms of the context.

	MA.912.AR.7 Solve and graph radical equations and functions in one and two variables.	MA.912.AR.7.1 Solve one-variable radical equations. Interpret solutions as viable in terms of context and identify any extraneous solutions. MA.912.AR.7.2 Given a table, equation or written description of a square root or cube root function, graph that function and determine its key features. MA.912.AR.7.3 Solve and graph mathematical and real-world problems that are modeled with square root or cube root functions. Interpret key features and determine constraints in terms of the context. MA.912.AR.7.4 Solve and graph mathematical and real-world problems that are modeled with radical functions. Interpret key features and determine constraints in terms of the context.
	MA.912.AR.8 Solve and graph rational equations and functions in one and two variables.	MA.912.AR.8.1 Write and solve one-variable rational equations. Interpret solutions as viable in terms of the context and identify any extraneous solutions. MA.912.AR.8.2 Given a table, equation or written description of a rational function, graph that function and determine its key features. MA.912.AR.8.3 Solve and graph mathematical and real-world problems that are modeled with rational functions. Interpret key features and determine constraints in terms of the context.
	MA.912.AR.9 Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.	MA.912.AR.9.1 Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically. MA.912.AR.9.2 Given a mathematical or real-world context, solve a system consisting of a two-variable linear equation and a non-linear equation algebraically or graphically. MA.912.AR.9.3 Given a mathematical or real-world context, solve a system consisting of two-variable linear or non-linear equations algebraically or graphically. MA.912.AR.9.4 Graph the solution set of a system of two-variable linear inequalities. MA.912.AR.9.5 Graph the solution set of a system of two-variable inequalities. MA.912.AR.9.6 Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options. MA.912.AR.9.7 Given a real-world context, represent constraints as systems of linear and non-linear equations or inequalities. Interpret solutions to problems as viable or non-viable options. MA.912.AR.9.8 Solve real-world problems involving linear programming in two variables. MA.912.AR.9.9 Given a mathematical or real-world context, solve a system of three-variable linear equations algebraically. MA.912.AR.9.10 Solve and graph mathematical and real-world problems that are modeled with piecewise functions. Interpret key features and determine constraints in terms of the context.

	MA.912.AR.10 Write and solve sequence and series equations, functions and inequalities in one and two variables.	MA.912.AR.10.1 Given a mathematical or real-world context, write and solve problems involving arithmetic sequences. MA.912.AR.10.2 Given a mathematical or real-world context, write and solve problems involving geometric sequences. MA.912.AR.10.3 Recognize and apply the formula for the sum of a finite arithmetic series to solve mathematical and real-world problems. MA.912.AR.10.4 Recognize and apply the formula for the sum of a finite or an infinite geometric series to solve mathematical and real-world problems. MA.912.AR.10.5 Given a mathematical or real-world context, write a sequence using function notation, defined explicitly or recursively, to represent relationships between quantities from a written description. MA.912.AR.10.6 Given a mathematical or real-world context, find the domain of a given sequence defined recursively or explicitly.