Mathematics for College Algebra B.E.S.T. Instructional Guide for Mathematics

The B.E.S.T. Instructional Guide for Mathematics (B1G-M) is intended to assist educators with planning for student learning and instruction aligned to Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards. This guide is designed to aid high-quality instruction through the identification of components that support the learning and teaching of the B.E.S.T. Mathematics Standards and Benchmarks. The B1G-M includes an analysis of information related to the B.E.S.T. Standards for Mathematics within this specific mathematics course, the instructional emphasis and aligned resources. This document is posted on the B.E.S.T. Standards for Mathematics webpage of the Florida Department of Education's website and will continue to undergo edits as needed.

Structural Framework and Intentional Design of the B.E.S.T. Standards for Mathematics

Florida's B.E.S.T. Standards for Mathematics were built on the following.

- The coding scheme for the standards and benchmarks was changed to be consistent with other content areas. The new coding scheme is structured as follows: Content.GradeLevel.Strand.Standard.Benchmark.
- Strands were streamlined to be more consistent throughout.
- The standards and benchmarks were written to be clear and concise to ensure that they are easily understood by all stakeholders.
- The benchmarks were written to allow teachers to meet students' individual skills, knowledge and ability.
- The benchmarks were written to allow students the flexibility to solve problems using a method or strategy that is accurate, generalizable and efficient depending on the content (i.e., the numbers, expressions or equations).
- The benchmarks were written to allow for student discovery (i.e., exploring) of strategies rather than the teaching, naming and assessing of each strategy individually.
- The benchmarks were written to support multiple pathways for success in career and college for students.
- The benchmarks should not be taught in isolation but should be combined purposefully.
- The benchmarks may be addressed at multiple points throughout the year, with the intention of gaining mastery by the end of the year.
- Appropriate progression of content within and across strands was developed for each grade level and across grade levels.
- There is an intentional balance of conceptual understanding and procedural fluency with the application of accurate real-world context intertwined within mathematical concepts for relevance.
- The use of other content areas, like science and the arts, within real-world problems should be accurate, relevant, authentic and reflect grade-level appropriateness.

Components of the B.E.S.T. Instructional Guide for Mathematics

The following table is an example of the layout for each benchmark and includes the defining attributes for each component. It is important to note that instruction should not be limited to the possible connecting benchmarks, related terms, strategies or examples provided. To do so would strip the intention of an educator meeting students' individual skills, knowledge and abilities.

Benchmark
focal point for instruction within lesson or task

This section includes the benchmark as identified in the B.E.S.T. Standards for Mathematics. The benchmark, also referred to as the Benchmark of Focus, is the focal point for student learning and instruction. The benchmark, and its related example(s) and clarification(s), can also be found in the course description. The 9-12 benchmarks may be included in multiple courses; select the example(s) or clarification(s) as appropriate for the identified course.

Connecting Benchmarks/Horizontal Alignment
in other standards within the grade level or course

This section includes a list of connecting benchmarks that relate horizontally to the Benchmark of Focus. Horizontal alignment is the intentional progression of content within a grade level or course linking skills within and across strands. Connecting benchmarks are benchmarks that either make a mathematical connection or include prerequisite skills. The information included in this section is not a comprehensive list, and educators are encouraged to find other connecting benchmarks. Additionally, this list will not include benchmarks from the same standard since benchmarks within the same standard already have an inherent connection.

Terms from the K-12 Glossary

This section includes terms from Appendix C: K-12 Glossary, found within the B.E.S.T. Standards for Mathematics document, which are relevant to the identified Benchmark of Focus. The terms included in this section should not be viewed as a comprehensive vocabulary list, but instead should be considered during instruction or act as a reference for educators.

Vertical Alignment
across grade levels or courses

This section includes a list of related benchmarks that connect vertically to the Benchmark of Focus. Vertical alignment is the intentional progression of content from one year to the next, spanning across multiple grade levels. Benchmarks listed in this section make mathematical connections from prior grade levels or courses in future grade levels or courses within and across strands. If the Benchmark of Focus is a new concept or skill, it may not have any previous benchmarks listed. Likewise, if the Benchmark of Focus is a mathematical skill or concept that is finalized in learning and does not have any direct connection to future grade levels or courses, it may not have any future benchmarks listed. The information included in this section is not a comprehensive list, and educators are encouraged to find other benchmarks within a vertical progression.

Purpose and Instructional Strategies

This section includes further narrative for instruction of the benchmark and vertical alignment. Additionally, this section may also include the following:

- explanations and details for the benchmark;
- vocabulary not provided within Appendix C;
- possible instructional strategies and teaching methods; and
- strategies to embed potentially related Mathematical Thinking and Reasoning Standards (MTRs).

Common Misconceptions or Errors

This section will include common student misconceptions or errors and may include strategies to address the identified misconception or error. Recognition of these misconceptions and errors enables educators to identify them in the classroom and make efforts to correct the misconception or error. This corrective effort in the classroom can also be a form of formative assessment within instruction.

Instructional Tasks
demonstrate the depth of the benchmark and the connection to the related benchmarks

This section will include example instructional tasks, which may be open-ended and are intended to demonstrate the depth of the benchmark. Some instructional tasks include integration of the Mathematical Thinking and Reasoning Standards (MTRs) and related benchmark(s). Enrichment tasks may be included to make connections to benchmarks in later grade levels or courses. Tasks may require extended time, additional materials and collaboration.

Instructional Items
demonstrate the focus of the benchmark

This section will include example instructional items which may be used as evidence to demonstrate the students' understanding of the benchmark. Items may highlight one or more parts of the benchmark.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Mathematical Thinking and Reasoning Standards

MTRs: Because Math Matters

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following.

- The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
- In order to fulfill Florida's unique coding scheme, the 5th place (benchmark) will always be a "1" for the MTRs.
- The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
- At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
- The bulleted language of the MTRs were written for students to use as self-monitoring tools during daily instruction.
- The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
- The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
- The MTRs should not stand alone as a separate focus for instruction, but should be combined purposefully.
- The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

Clarifications:
Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:
Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:
Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

Clarifications:
Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students' ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:
Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:
Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:
Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

Examples of Teacher and Student Moves for the MTRs

Below are examples that demonstrate the embedding of the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction keeping in mind the benchmark(s) that are the focal point of the lesson or task. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

MTR	**Student Moves**	**Teacher Moves**
MA.K12.MTR.1.1 *Actively participate in effortful learning both individually and collectively.*	• Students engage in the task through individual analysis, student-to-teacher interaction and student-to-student interaction. • Students ask task-appropriate questions to self, the teacher and to other students. *(MTR.4.1)* • Students have a positive productive struggle exhibiting growth mindset, even when making a mistake. • Students stay engaged in the task to a purposeful conclusion while modifying methods, when necessary, in solving a problem through self-analysis and perseverance.	• Teacher provides flexible options (i.e., differentiated, challenging tasks that allow students to actively pursue a solution both individually and in groups) so that all students have the opportunity to access and engage with instruction, as well as demonstrate their learning. • Teacher creates a physical environment that supports a growth mindset and will ensure positive student engagement and collaboration. • Teacher provides constructive, encouraging feedback to students that recognizes their efforts and the value of analysis and revision. • Teacher provides appropriate time for student processing, productive struggle and reflection. • Teacher uses data and questions to focus students on their thinking; help students determine their sources of struggle and to build understanding. • Teacher encourages students to ask appropriate questions of other students and of the teacher including questions that examine accuracy. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.2.1 *Demonstrate understanding by representing problems in multiple ways.*	• Students represent problems concretely using objects, models and manipulatives. • Students represent problems pictorially using drawings, models, tables and graphs. • Students represent problems abstractly using numerical or algebraic expressions and equations. • Students make connections and select among different representations and methods for the same problem, as appropriate to different situations or context. *(MTR.3.1)*	• Teacher provides students with objects, models, manipulatives, appropriate technology and real-world situations. *(MTR.7.1)* • Teacher encourages students to use drawings, models, tables, expressions, equations and graphs to represent problems and solutions. • Teacher questions students about making connections between different representations and methods and challenges students to choose one that is most appropriate to the context. *(MTR.3.1)* • Teacher encourages students to explain their different representations and methods to each other. *(MTR.4.1)* • Teacher provides opportunities for students to choose appropriate methods and to use mathematical technology.
MA.K12.MTR.3.1 *Complete tasks with mathematical fluency.*	• Students complete tasks with flexibility, efficiency and accuracy. • Students use feedback from peers and teachers to reflect on and revise methods used. • Students build confidence through practice in a variety of contexts and problems. *(MTR.1.1)*	• Teacher provides tasks and opportunities to explore and share different methods to solve problems. *(MTR.1.1)* • Teacher provides opportunities for students to choose methods and reflect (i.e., through error analysis, revision, summarizing methods or writing) on the efficiency and accuracy of the method(s) chosen. • Teacher asks questions and gives feedback to focus student thinking to build efficiency of accurate methods. • Teacher offers multiple opportunities to practice generalizable methods.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.4.1 *Engage in discussions that reflect on the mathematical thinking of self and others.*	• Students use content specific language to communicate and justify mathematical ideas and chosen methods. • Students use discussions and reflections to recognize errors and revise their thinking. • Students use discussions to analyze the mathematical thinking of others. • Students identify errors within their own work and then determine possible reasons and potential corrections. • When working in small groups, students recognize errors of their peers and offers suggestions.	• Teacher provides students with opportunities (through open-ended tasks, questions and class structure) to make sense of their thinking. *(MTR.1.1)* • Teacher uses precise mathematical language, both written and abstract, and encourages students to revise their language through discussion. • Teacher creates opportunities for students to discuss and reflect on their choice of methods, their errors and revisions and their justifications. • Teachers select, sequence and present student work to elicit discussion about different methods and representations. *(MTR.2.1, MTR.3.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.5.1 *Use patterns and structure to help understand and connect mathematical concepts.*	• Students identify relevant details in a problem in order to create plans and decompose problems into manageable parts. • Students find similarities and common structures, or patterns, between problems in order to solve related and more complex problems using prior knowledge.	• Teacher asks questions to help students construct relationships between familiar and unfamiliar problems and to transfer this relationship to solve other problems. *(MTR.1.1)* • Teacher provides students opportunities to connect prior and current understanding to new concepts. • Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. *(MTR.3.1, MTR.4.1)* • Teacher allows students to develop an appropriate sequence of steps in solving problems. • Teacher provides opportunities for students to reflect during problem solving to make connections to problems in other contexts, noticing structure and making improvements to their process.
MA.K12.MTR.6.1 *Assess the reasonableness of solutions.*	• Students estimate a solution, including using benchmark quantities in place of the original numbers in a problem. • Students monitor calculations, procedures and intermediate results during the process of solving problems. • Students verify and check if solutions are viable, or reasonable, within the context or situation. *(MTR.7.1)* • Students reflect on the accuracy of their estimations and their solutions.	• Teacher provides opportunities for students to estimate or predict solutions prior to solving. • Teacher encourages students to compare results to estimations and revise if necessary for future situations. *(MTR.5.1)* • Teacher prompts students to self-monitor by continually asking, “Does this solution or intermediate result make sense? How do you know?” • Teacher encourages students to provide explanations and justifications for results to self and others. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.*	• Students connect mathematical concepts to everyday experiences. • Students use mathematical models and methods to understand, represent and solve real-world problems. • Students investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Students re-design models and methods to improve accuracy or efficiency.	• Teacher provides real-world context to help students build understanding of abstract mathematical ideas. • Teacher encourages students to assess the validity and accuracy of mathematical models and situations in real-world context, and to revise those models if necessary. • Teacher provides opportunities for students to investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Teacher provides opportunities for students to apply concepts to other content areas.

Mathematics for College Algebra Areas of Emphasis

In Mathematics for College Algebra, instructional time will emphasize five areas:

(1) developing fluency with the Laws of Exponents with numerical and algebraic expressions;
(2) extending arithmetic operations with algebraic expressions to include rational and polynomial expressions;
(3) solving one-variable exponential, logarithmic, radical and rational equations and interpreting the viability of solutions in real-world contexts;
(4) modeling with and applying linear, quadratic, absolute value, exponential, logarithmic and piecewise functions and systems of linear equations and inequalities; and
(5) extending knowledge of functions to include inverse and composition

The purpose of the areas of emphasis is not to guide specific units of learning and instruction, but rather provide insight on major mathematical topics that will be covered within this mathematics course. In addition to its purpose, the areas of emphasis are built on the following.

- Supports the intentional horizontal progression within the strands and across the strands in this grade level or course.
- Student learning and instruction should not focus on the stated areas of emphasis as individual units.
- Areas of emphasis are addressed within standards and benchmarks throughout the course so that students are making connections throughout the school year.
- Some benchmarks can be organized within more than one area.
- Supports the communication of the major mathematical topics to all stakeholders.
- Benchmarks within the areas of emphasis should not be taught within the order in which they appear. To do so would strip the progression of mathematical ideas and miss the opportunity to enhance horizontal progressions within the grade level or course.

The table on the next page shows how the benchmarks within this mathematics course are embedded within the areas of emphasis.

Domain	Standard	Properties of exponents and logarithms	Operations with algebraic expressions	Solving one-variable equations	Modeling functions and systems of equations	Inverse and composition functions
Number Sense and Operations	MA.912.NSO.1.1	x		x		x
	MA.912.NSO.1.2	x				
	MA.912.NSO.1.3	x				
	MA.912.NSO.1.6	x		x		x
	MA.912.NSO.1.7	x	x			x
Algebraic Reasoning	MA.912.AR.1.2	x		x		
	MA.912.AR.1.3		x			x
	MA.912.AR.1.5		x			x
	MA.912.AR.1.9		x	x		
	MA.912.AR.2.4				x	
	MA.912.AR.2.5			x	x	
	MA.912.AR.3.7				x	
	MA.912.AR.3.8			x	x	
	MA.912.AR.4.2				x	
	MA.912.AR.4.4			x	x	
	MA.912.AR.5.2	x		x		
	MA.912.AR.5.4	x			x	
	MA.912.AR.5.6	x		x	x	x
	MA.912.AR.5.7	x		x	x	x
	MA.912.AR.5.8	x		x	x	x
	MA.912.AR.5.9	x		x	x	x
	MA.912.AR.7.1			x	x	
	MA.912.AR.8.1		x	x	x	
	MA.912.AR.9.4				x	
	MA.912.AR.9.6				x	
	MA.912.AR.9.10		x		x	
Functions	MA.912.F.1.1					x
	MA.912.F.1.2					x
	MA.912.F.1.3				x	x
	MA.912.F.1.6	x				x
	MA.912.F.2.1					x
	MA.912.F.2.2					x
	MA.912.F.2.3					x
	MA.912.F.2.4					x
	MA.912.F.2.5					x
	MA.912.F.3.2		x		x	x
	MA.912.F.3.4					x
	MA.912.F.3.6	x				x
	MA.912.F.3.7	x				x

MA.912.NSO.1 *Generate equivalent expressions and perform operations with expressions involving exponents, radicals or logarithms.*

MA.912.NSO.1.1

Benchmark

MA.912.NSO.1.1	Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.

Benchmark Clarifications:
Clarification 1: Instruction includes the use of technology when appropriate.
Clarification 2: Refer to the K-12 Formulas (Appendix E) for the Laws of Exponents.
Clarification 3: Instruction includes converting between expressions involving rational exponents and expressions involving radicals.
Clarification 4: Within the Mathematics for Data and Financial Literacy course, it is not the expectation to generate equivalent numerical expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.2, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7
- MA.912.F.1.2, MA.912.F.1.6

Terms from the K-12 Glossary

- Base
- Expression
- Exponent

Vertical Alignment

Previous Benchmarks

- MA.8.NSO.1.3

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rational-number exponents to generate and evaluate numerical expressions (MA.912.NSO.1.1). In Mathematics for College Algebra, students extend the Laws of Exponents to properties of logarithms.

- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.
- Instruction includes a continuation from Algebra I of working with patterns and the connection to mathematical operations and the inverse relationship between powers and radicals (*MTR.5.1*).
- Students should make the connection of the root being equivalent to a unit fraction exponent *(MTR.4.1)*.
    - For example, $\sqrt[3]{8} = \sqrt[3]{2^3}$ is equivalent to the equation $\sqrt[3]{8} = (2^3)^{\frac{1}{3}}$, which is equivalent to the equation $\sqrt[3]{8} = 2^{\frac{3}{1} \cdot \frac{1}{3}}$, which is equivalent to the equation $\sqrt[3]{8} = 2^1$, which is equivalent to the equation $\sqrt[3]{8} = 2$.
- Students should be able to fluently apply the Laws of Exponents in both directions.
    - For example, students should recognize that 2^6 is the quantity $(2^3)^2$.

- Instruction includes students being able to fluently evaluate numerical expressions with and without the use of technology.

Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may try to perform operations on bases as well as exponents.
- Students may multiply the base by the exponent instead of understanding that the exponent is the number of times the base occurs as a factor.
- Students may not truly understand exponents that are zero or negative.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

Part A. Natasha believes that any number to the zero power is one, and zero to any power is zero. Is her thinking correct? What would 0^0 be equivalent to?

Part B. Louis enters 0^{-2} on his calculator and receives an ERROR message. Explain to Louis why he received this message.

Instructional Task 2 (MTR.3.1, MTR.4.1)

Evaluate the following without using the use of technology. Answers should only have a single power and can expressed as a fraction or decimal.

Part A. $\frac{7 \cdot 8^{2022}}{3 \cdot 8^{2024}}$

Part B. $\frac{(11^{74})^{10}}{(11^{82})^{9}}$

Instructional Task 3 (MTR.4.1, MTR.5.1)

Part A. Given $f(x) = 1.5^x$, complete the table below without the use of a calculator. Share with a partner how you determine the value of $f(x)$ for each.

$f(x)$								
x	4	3	2	1	0	-1	-2	-3

Part B. Graph the function f in the domain of $-3 \le x \le 4$ using technology. How does this compare to your table of values in Part A?

Part C. Using your graph from Part B, estimate to one decimal place, the value of x when $f(x) = 2$.

Instructional Items

Instructional Item 1

Verify whether the equation below is true or false.

$$(4 \cdot 9)^5 = 4^5 \cdot 9^5$$

Instructional Item 2

Evaluate the numerical expression $\left(-\frac{729}{64}\right)^{-\frac{2}{3}}$.

Instructional Item 3

Choose all of the expressions that are equivalent to $7^{\frac{5}{12}}$

a. $(49^{\frac{1}{3}})(7^{-\frac{1}{4}})$

b. $(7^{\frac{2}{3}})(7^{-\frac{1}{4}})$
c. $7(7^{-\frac{1}{4}})$
d. $\sqrt[5]{7^{12}}$
e. $\sqrt[12]{7^{5}}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.2

Benchmark

MA.912.NSO.1.2 Generate equivalent algebraic expressions using the properties of exponents.

> *Example:* The expression 1.5^{3t+2} is equivalent to the expression $2.25(1.5)^{3t}$ which is equivalent to $2.25(3.375)^{t}$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.2, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7
- MA.912.F.1.2, MA.912.F.1.6

Terms from the K-12 Glossary

- Base
- Exponent
- Expression

Vertical Alignment

Previous Benchmarks

- MA.8.AR.1.1

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students generated equivalent algebraic expressions using the Laws of Exponents with integer exponents. In Algebra I, students expanded this work to include rational-number exponents. In Math for College Algebra, students extend the Laws of Exponents to algebraic expressions with logarithms.

- Instruction of this benchmark helps build the foundation for operations with exponents and logarithms throughout the course.
- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.
- Instruction includes student discovery of the patterns and the connection to mathematical operations and the inverse relationship between powers and radicals *(MTR.5.1)*.
- Students should be able to fluently apply the Laws of Exponents in both directions.
  - For example, students should recognize that a^6 is the quantity $(a^3)^2$; this may be helpful when students are factoring a difference of squares.
- When generating equivalent expressions, students should be encouraged to approach from different entry points and discuss how they are different but equivalent strategies.

Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may not have fully mastered the Laws of Exponents and understand the mathematical connections between the bases and the exponents.

- Student may believe that with the introduction of variables, the properties of exponents differ from numerical expressions.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.5.1)

What value of x will create a true statement in the equation $144^x = 1728$?

Instructional Task 2 (MTR.3.1, MTR.4.1)

Part A. Write the algebraic expression $\left(\frac{6x^{\frac{2}{7}}y^{-4}z^0}{9x^2y^5z^{-8}}\right)^3$, as an equivalent expression where each variable only appears once.

Part B. Compare your method of simplifying with a partner.

Instructional Items

Instructional Item 1

Given the algebraic expression 2.3^{2t-1}, create an equivalent expression.

Instructional Item 2

Use the properties of exponents to create an equivalent expression for the given expression shown below with no variables in the denominator.

$$(4x^{-0.5})^3 \div (9x^{\frac{1}{3}})^{-1.5}$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.3

Benchmark

MA.912.NSO.1.3	Generate equivalent algebraic expressions involving radicals or rational exponents using the properties of exponents.

Benchmark Clarifications:

Clarification 1: Within the Algebra 2 course, radicands are limited to monomial algebraic expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.2, MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7
- MA.912.AR.7.1
- MA.912.F.1.2, MA.912.F.1.6

Terms from the K-12 Glossary

- Base
- Exponent
- Expression
- Rational Number

Vertical Alignment

Previous Benchmarks

- MA.8.AR.1.1

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students generated equivalent algebraic expressions with rational-number exponents and performed operations with numerical expressions involving square or cube roots.

In Math for College Algebra, students extend the Laws of Exponents to algebraic expressions involving radicals.

- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.
- Instruction includes student discovery of the patterns and the connection to mathematical operations *(MTR.5.1)*.
- Students should be able to fluently apply the Laws of Exponents in both directions.
    - For example, students should recognize that a^6 is the quantity $(a^3)^2$, this may be helpful when students are factoring a difference of squares.
- When generating equivalent expressions, students should be encouraged to approach from different entry points and discuss how they are different but equivalent strategies *(MTR.2.1)*.
- It is important to reinforce and activate the prior knowledge of simple calculations with radicals within this benchmark.

Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may not have fully mastered the Laws of Exponents and understand the mathematical connections between the bases and the exponents.
- Student may believe that with the introduction of variables, the properties of exponents differ from numerical expressions.
- Students may not know how to do simple calculations with radicals; therefore, they may not take the square root of the perfect square factor, or they may suggest using a factor pair within a radical that does not contain a perfect square as a factor.
- Students may confuse radicands and coefficients and perform the operations on the wrong part of the expression.
    - For example, express $(2p)^{\frac{1}{5}}$ in radical form. The correct answer is $\sqrt[5]{2p}$ instead of $2\sqrt[5]{p}$.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.3.1, MTR.5.1)

Evaluate the expression $\sqrt{\sqrt[3]{64}}$. Compare your strategy with a partner.

Instructional Task 2 (MTR.2.1, MTR.3.1, MTR.5.1)

Part A. Without the use of technology, graph $f(x) = \sqrt[3]{x}$ over the domain $-1 \leq x \leq 1$.

Part B. Without the use of technology, graph $f(x) = x^{\frac{2}{3}}$ over the domain $-1 \leq x \leq 1$.

Part C. Compare the graphs from Part A and Part B.

Instructional Items

Instructional Item 1

Express the following as a radical $(36x^4)^{0.5}$.

Instructional Item 2

Expression $5\sqrt[3]{abc}$ as an expression with exponents.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Benchmark

MA.912.NSO.1.6 Given a numerical logarithmic expression, evaluate and generate equivalent numerical expressions using the properties of logarithms or exponents.

Benchmark Clarifications:
Clarification 1: Within the Mathematics for Data and Financial Literacy Honors course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.2, MA.912.AR.5.8, MA.912.AR.5.9

Terms from the K-12 Glossary

- Base
- Exponent
- Expression
- Inverse function

Vertical Alignment

Previous Benchmarks

- MA.8.NSO.1.3
- MA.912.AR.5.3

Next Benchmarks

- MA.912.C.2.3

Purpose and Instructional Strategies

In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rational-number exponents. In Mathematics for College Algebra, students extend the Laws of Exponents to the properties of logarithms.

- Instruction makes the connection between the properties of logarithms and the properties of exponents. Explain that a logarithm is defined as an exponent establishing the equivalence of $y = a^x$ and, $x = \log_a y$ given $a > 0$ and $a \neq 1$ *(MTR.5.1)*. Remind students that logarithmic and exponential operations are inverse operations, so the properties of the logarithms are the “opposite” of the properties of the exponents.

Properties of Exponents		Properties of Logarithms	
$a^m \cdot a^n = a^{m+n}$	$\rightarrow$	$\log_a(m \cdot n) = \log_a m + \log_a n$	Product Property
$\dfrac{a^m}{a^n} = a^{m-n}$	$\rightarrow$	$\log_a\left(\dfrac{m}{n}\right) = \log_a m - \log_a n$	Quotient Property
$(a^m)^n = a^{mn}$	$\rightarrow$	$\log_a(m)^n = n(\log_a m)$	Power Property
		$\log_a(m) = \dfrac{\log_b m}{\log_b a} = \dfrac{\log m}{\log a} = \dfrac{\ln m}{\ln a}$	Change of Base
		Other Properties	
$a^0 = 1$	$\rightarrow$	$\log_a 1 = 0$	
$a^1 = a$	$\rightarrow$	$\log_a a = 1$	

- Instruction includes making the connection between the change of base formula and the inverse relationship between exponents and logarithms.
    - For example, students should know that by definition $b^{\log_b x} = x$. Therefore, students can take the log with base a of both sides of the equation to obtain $\log_a b^{\log_b x} = \log_a x$. Then, students can use the the Power Property to rewrite

the equation as $(\log_b x)(\log_a b) = \log_a x$. Students should notice that there the arguments for two of the logs are b and x with each the same base of a. So, one can divide both sides of the equation by $\log_a b$ to isolate $\log_b x$, obtaining $\frac{(\log_b x)(\log_a b)}{(\log_a b)} = \frac{\log_a x}{(\log_a b)}$. Therefore, $\log_b x = \frac{\log_a x}{\log_a b}$.

- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate them or use the properties *(MTR.4.1)*.
  - For example, present students with $\log_{1.03} 1.092727$ and ask them for its meaning, "the exponent required on the base 1.03 to obtain 1.092727."
- Many examples in this course will require students to utilize the change of base formula to evaluate logarithms. Consider the example above.

$$\log_{1.03} 1.092727 = \frac{\log 1.092727}{\log 1.03} \text{ or } \frac{\ln 1.092727}{\ln 1.03}$$

  - Evaluating this expression with a calculator requires the use of common logs or natural logs. Guide students to understand that common logs have a base of 10 while natural logs have a base of e and are easy to enter into a calculator. Have students use a calculator to evaluate the expression to find it is approximately equivalent to 3. Have students confirm this by checking the exponential equivalent expression $1.03^3 = 1.092727$.
  - While students will see more complex logarithms in this course, it is appropriate to simplify the logarithms they work with initially in this benchmark until they reach an understanding of the properties of logarithms. Depending on the courses students have taken prior to Math for College Algebra, this may their first introduction to logarithms.
- Instruction includes the understanding that logarithms with base 10 are called common logarithms and written as $\log x$ or $\log_{10} x$.
- Instruction includes the understanding that logarithms with base e are called natural logarithms and written as $\ln x$ or $\log_e x$.
- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate it or use the properties.
  - For example, present students with $\log_2 64$ and ask for its meaning, "the exponent required on the base 2 to obtain 64" *(MTR.2.1)*.

- Students should understand how to use their one-to-one and inverse properties to evaluate expressions by making decisions on which base is best to use for solving.
  - One-to-One Properties

$a^y = a^x$	$y = x$
$log_a x = log_a y$	$x = y$

  - Inverse Properties

Logarithmic Form	**Exponential Form**
$log_a 1 = 0$	$a^0 = 1$
$log_a a = 1$	$a^1 = a$
$log_a a^x = x$	$a^x = a^x$
$a^{log_a x} = x$	$a^{x^x} = a^x$

- Students should have practice combining the properties of logarithms to generate equivalent algebraic expressions by either simplifying (condensing) or expanding the

expression.
- Problem types include logarithms with different bases, including common logarithms and natural logarithms.

Common Misconceptions or Errors

- Students may see "log" as a variable rather than as an operation.
  - For example, they may see "log" as a common factor in the expression, $\log 12 + \log 6$ and mistakenly write $\log(12 + 6)$.
  - For example, they may rewrite the expression $\log(12 \cdot 6)$ and as $\log 12 \cdot \log 6$ instead of $\log 12 + \log 6$. Similarly, they may incorrectly rewrite $\log\left(\frac{12}{6}\right)$ as $\frac{\log 12}{\log 6}$ instead of $\log 12 - \log 6$.
  - For example, they may cancel the "log" from the numerator and the denominator in an expression.
- Students may cancel the "log" from the numerator and the denominator in an expression. Remind students that just like with roots we can simplify or expand logarithms if the argument is fully factored.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Recall that $\log_b(x)$ is by definition the exponent which b must be raised to in order to yield x $(b > 0)$.

Part A.
- Use this definition to compute $\log_2(2^5)$.
- Use this definition to compute $\log_{10}(0.001)$.
- Use this definition to compute $\ln(e^3)$.
- Explain why $\log_b(b^y) = y$ where $b > 0$.

Part B.
- Evaluate $10^{\log_{ln}(100)}$.
- Evaluate $2^{\log_2(\sqrt{2})}$.
- Evaluate $e^{\ln(89)}$.
- Explain why $b^{\log_b x} = x$ where $b > 0$.

Instructional Items

Instructional Item 1 (MTR.3.1)

Use the properties of logarithms to rewrite each expression into lowest terms.

a. $\log_2 2^5$

b. $\log_3 \frac{12}{5}$

c. $\log_5 3 + \log_5 13$

Instructional Item 2

What is the value of the logarithmic expression $\log_2(2^6)$?

Instructional Item 3

Which of the following expressions are equivalent to $\log_4 16$? Select all that apply.

a. $\frac{\log 4}{\log 16}$

b. $\frac{\log 16}{\ln 4}$

c. $\frac{\ln 4}{\ln 16}$

d. $\frac{\ln 16}{\ln 4}$

e. $\frac{\ln 4}{\log 16}$

f. $\frac{\log 16}{\log 4}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.7

Benchmark

MA.912.NSO.1.7 Given an algebraic logarithmic expression, generate an equivalent algebraic expression using the properties of logarithms or exponents.

Benchmark Clarifications:
Clarification 1: Within the Mathematics for Data and Financial Literacy Honors course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.2, MA.912.AR.5.8, MA.912.AR.5.9

Terms from the K-12 Glossary

- Inverse function

Vertical Alignment

Previous Benchmarks

- MA.8.NSO.1.3

Next Benchmarks

- MA.912.C.2.8

Purpose and Instructional Strategies

In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rational-number exponents. In Mathematics for College Algebra, students extend the Laws of Exponents to the Properties of Logarithms.

- Instruction makes the connection between the properties of logarithms and the properties of exponents. Explain that a logarithm is defined as an exponent establishing the equivalence of $y = a^x$ and, $x = \log_a y$ given $a > 0$ and $a \neq 1$ *(MTR.5.1)*. Remind students that logarithmic and exponential operations are inverse operations, so the properties of the logarithms are the "opposite" of the properties of the exponents.

Properties of Exponents		Properties of Logarithms	
$a^m \cdot a^n = a^{m+n}$	$\rightarrow$	$\log_a(m \cdot n) = \log_a m + \log_a n$	Product Property
$\frac{a^m}{a^n} = a^{m-n}$	$\rightarrow$	$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$	Quotient Property
$(a^m)^n = a^{mn}$	$\rightarrow$	$\log_a(m)^n = n(\log_a m)$	Power Property

		$\log_a(m) = \frac{\log_b m}{\log_b a} = \frac{\log m}{\log a} = \frac{\ln m}{\ln a}$	Change of Base
		Other Properties	
$a^0 = 1$	$\rightarrow$	$\log_a 1 = 0$	
$a^1 = a$	$\rightarrow$	$\log_a a = 1$	

- Instruction includes making the connection between the change of base formula and the inverse relationship between exponents and logarithms.
    - For example, students should know that by definition $b^{\log_b x} = x$. Therefore, students can take the log with base a of both sides of the equation to obtain $\log_a b^{\log_b x} = \log_a x$. Then, students can use the the Power Property to rewrite the equation as $(\log_b x)(\log_a b) = \log_a x$. Students should notice that there the arguments for two of the logs are b and x with each the same base of a. So, one can divide both sides of the equation by $\log_a b$ to isolate $\log_b x$, obtaining $\frac{(\log_b x)(\log_a b)}{(\log_a b)} = \frac{\log_a x}{(\log_a b)}$. Therefore, $\log_b x = \frac{\log_a x}{\log_a b}$.
- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate them or use the properties *(MTR.4.1)*.
    - For example, present students with $\log_{1.03} 1.092727$ and ask them for its meaning, "the exponent required on the base 1.03 to obtain 1.092727."
- Students should have practice combining the properties of logarithms to generate equivalent algebraic expressions by either simplifying (condensing) or expanding the expression.
- Problem types include logarithms with different bases, including common logarithms and natural logarithms.

Common Misconceptions or Errors

- Students tend to treat "log" as a variable rather than as an operation:
  - For example, they may see "log" as a common factor in the expression $\log x + \log y$ and mistakenly write $\log(x + y)$.
  - For example, they may distribute the "log" in the expression $\log(a \cdot b)$ and rewrite it as $\log a \cdot \log b$ instead of $\log_a m + \log_a n$. Similarly, they may incorrectly rewrite $\log\left(\frac{a}{b}\right)$ as $\frac{\log a}{\log b}$.
  - For example, they may divide both sides of the equation $\log(7x - 12) = 2\log x$ by "log" to mistakenly obtain $7x - 12 = 2x$.
- Students may cancel the "log" from the numerator and the denominator in an expression. Remind students that just like with roots we can simplify or expand logarithms if the argument is fully factored.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

The Loudness of Sound formula, measure in decibels (dB), is $L = 10\ \log I$, where L is the loudness, and I is the intensity of sound.

Part A. The formula $\Delta L = L_2 - L_1$, describe the sound intensity level between two loudness, L_1 and L_2. Write the formula in terms of the intensity of the sounds, I_2 and I_1.

Part B. Rewrite the formula for the sound intensity level, ΔL, as a single logarithm.

Part C. The table below shows the Ratios of Intensities and Corresponding Differences in Sound Intensity Levels. Using the Properties of Logarithms show that if a sound is 100 times as intense as another, it has a sound level about 20 dB higher.

I_2/I_1	$L_2 - L_1$
2.0	3.0 dB
5.0	7.0 dB
10.0	10.0 dB
100.0	20.0 dB
1000.0	30.0 dB

Part D. If the lowest or threshold intensity of sound a person with normal hearing can perceive is $I_0 = 10^{-12}\ W/m^2$, what is the intensity of sound at a pain level of 120 dB?

Instructional Task 2 (MTR.4.1, MTR.5.1)

Use the functions below to answer the following questions.

$$f(x) = 10^{0.2x}$$
$$h(x) = 5(\log x)$$

Part A. Use technology to graph the functions f and h on the same coordinate plane. What do you notice?

Part B. Determine $f(2)$ and $h(2)$. What do you notice?

Part C. Discuss with a partner why logarithmic functions are said to be the inverse of exponential functions.

Instructional Items

Instructional Item 1

Use the properties of logarithms to rewrite each expression into lowest terms.

Part A. $\log_4 4x^2$

Part B. $\ln \frac{xy}{z}$

Instructional Item 2

Write each expression as a single logarithmic quantity:

Part A. $3 \ln x + 4\ln y - 5 \ln z$

Part B. $\frac{3}{2}\log_2 x^6 - \frac{3}{4}\log_2 x^8$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1 *Interpret and rewrite algebraic expressions and equations in equivalent forms.*

MA.912.AR.1.2

Benchmark

MA.912.AR.1.2 Rearrange equations or formulas to isolate a quantity of interest.

Algebra I Example: The Ideal Gas Law $PV = nRT$ can be rearranged as $T = \frac{PV}{nR}$ to isolate temperature as the quantity of interest.

Example: Given the Compound Interest formula $= P(1 + \frac{r}{n})^{nt}$, solve for P.

Mathematics for Data and Financial Literacy Honors Example: Given the Compound Interest formula $A = P(1 + \frac{r}{n})^{nt}$, solve for t.

Benchmark Clarifications:

Clarification 1: Instruction includes using formulas for temperature, perimeter, area and volume; using equations for linear (standard, slope-intercept and point-slope forms) and quadratic (standard, factored and vertex forms) functions.

Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.2, MA.912.NSO.1.3, MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.2, MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.7.1
- MA.912.AR.8.1

Terms from the K-12 Glossary

- Equation

Vertical Alignment

Previous Benchmarks

- MA.8.AR.2.3
- MA.912.AR.2.1
- MA.912.AR.3.1
- MA.912.AR.4.1

Next Benchmarks

- MA.912.AR.6.1, MA.912.AR.6.6
- MA.912.AR.7.3, MA.912.AR.7.4
- MA.912.AR.8.3

Purpose and Instructional Strategies

In grade 8, students isolated variables in equations in the form $x^2 = p$ and $x^3 = q$. In Algebra I, students isolated a variable or quantity of interest in linear, absolute value and quadratic equations and formulas. In Mathematics for College Algebra, students highlight a variable or quantity of interest for other types of equations and formulas, including exponential, logarithmic, radical, and rational.

- Instruction includes making connections to inverse arithmetic operations (refer to Appendix D) and solving one-variable equations.
- Instruction includes making connections to inverse functions, such as, exponential and logarithm, square and square root, raising to the nth power and taking the nth root.
- Instruction includes justifying each step while rearranging an equation or formula.
  - For example, when rearranging $A = P\left(1 + \frac{r}{n}\right)^{nt}$ for P, it may be helpful for students to highlight the quantity of interest with a highlighter, so students remain focused on that quantity for isolation purposes. It may also be helpful for students to identify factors or other parts of the equations.

Common Misconceptions or Errors

- Students may not have mastered the inverse arithmetic operations.
- Students may not see that inverse operations can be performed with groups of numbers and variables treated as a single part of the formula (MA.912.AR.1.1).
- Students may be frustrated because they are not arriving at a numerical value as their solution. Remind students that they are rearranging variables that can be later evaluated as a numerical value.
- Having multiple variables and no values may confuse students and make it difficult for them to see the connections between rearranging a formula and solving one-variable equations.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

Part A. Given the equation $ax^2 + bx + c = 0$, solve for x.

Part B. Share your strategy with a partner. What do you notice about the new equation(s)?

Instructional Task 2 (MTR.7.1)

The period of a simple pendulum is calculated using the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum and g is the acceleration due to gravity.

Part A. Solve the formula for L, the length of the pendulum.

Part B. A pendulum on Earth, $g = 9.80\frac{m}{s^2}$, has a period of $0.6\ s$, find its length.

Part C. Solve the formula for g, the acceleration of gravity.

Part D. If the same pendulum is moved to the moon, its period will be approximately $1.47\ s$. Calculate acceleration of gravity on the moon.

Instructional Task 3 (MTR.7.1)

The total resistance, R_T, for a parallel combination of n resistors is found using the formula $R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots\frac{1}{R_n}}$.

Part A. If the total resistance, R_T, for a parallel circuit with two resistors is 1.71Ω and one of the resistors has a resistance of 3Ω, what is the resistance of the second resistor?

Part B. If we want to reduce the total resistance, R_T, of the parallel circuit to 1.5Ω by adding a third resistor, R_3, what should be the resistance of R_3?

Instructional Items

Instructional Item 1

The formula for the velocity of a satellite moving around Earth is $= \sqrt{\frac{6.67\times10^{-11}(m)}{r}}$, where m is the mass of the central body about which the satellite orbits, in this case Earth and r is the radius of orbit for the satellite. Solve for the radius r.

Instructional Item 2

The lens equation $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ expresses the quantitative relationship among the distance, d_o, between an object and a lens, the distance, d_i, between the lens and the image position, and the focal length, f, of the lens. Solve for the distance between the lens and the image, d_i.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.3

Benchmark

MA.912.AR.1.3 Add, subtract and multiply polynomial expressions with rational number coefficients.

Benchmark Clarifications:
Clarification 1: Instruction includes an understanding that when any of these operations are performed with polynomials the result is also a polynomial.
Clarification 2: Within the Algebra I course, polynomial expressions are limited to 3 or fewer terms.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1, MA.912.NSO.1.2
- MA.912.F.3.2

Terms from the K-12 Glossary

- Polynomial

Vertical Alignment

Previous Benchmarks

- MA.7.AR.1.1
- MA.8.AR.1.2, MA.8.AR.1.3
- MA.912.AR.1.7

Next Benchmarks

- MA.912.AR.6.3

Purpose and Instructional Strategies

In middle grades, students added, subtracted and multiplied linear expressions. In Algebra I, students performed operations on polynomials limited to 3 or fewer terms. In Mathematics for College Algebra, students perform operations on all polynomials.

- Instruction includes making the connection to dividing polynomials and the understanding that division does not have closure.
- Reinforce like terms during instruction (using different colors can be a strategy to help identify them as unique from one another).
- Instruction includes the use of manipulatives, like algebra tiles, and various strategies, like the area model, properties of exponents and the distributive property *(MTR.2.1)*.
    - Area Model
      The expression $(2x^2 + 1.5x + 6)(3x + 4.2)$ is equivalent to $6x^3 + 12.9x^2 + 24.3x + 25.2$ and can be modeled below:

	$2x^2$	$1.5x$	6
$3x$	$6x^3$	$4.5x^2$	$18x$
4.2	$8.4x^2$	$6.3x$	25.2

- Instruction should not rely upon the use of tricks or acronyms, like FOIL.

Common Misconceptions or Errors

- Students may not understand the meaning of closure or the operations it applies to with polynomials.
- Students may not understand like terms or the properties of exponents.
- Students might think that polynomials are limited to monomials, binomials, and trinomials with just one variable. Provide practice with polynomials with multiple variables and multiple terms.
- Students may not correctly apply the distributive property when multiplying polynomials, leaving out one or more terms. Encourage the use of the area model so that is easier for them to account for all the terms.
- Students may incorrectly distribute the exponent when expanding a binomial raised to a power. Remind the students that, the exponent indicates how many times the based is multiplied.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

Part A. Determine the sum of $3x^2 - 2x + 5$ and $\frac{1}{6}x^2 + 7x + \frac{8}{7}$. Explain the method used in determining the sum.

Part B. Discuss whether the addition of polynomials will always result in another polynomial. Why or why not?

Part C. Determine the difference of $3x^2 - 2x + 5$ and $x^2 - 0.25x + 1.24$. Explain the method used in determining the difference.

Part D. Discuss whether the subtraction of polynomials will always result in another polynomial. Why or why not?

Part E. Determine the product of $2x + 5$ and $\frac{2}{9}x^2 - \frac{11}{2}x + 1$. Explain the method used in determining the product.

Part F. Discuss whether the multiplication of polynomials will always result in another polynomial. Why or why not?

Part G. Determine the quotient of $9x^2 - 3x + 12$ and $3x$. Explain the method used in determining the quotient.

Part H. Discuss whether the division of polynomials will always result in another polynomial. Why or why not?

Instructional Items

Instructional Item 1

Determine the sum/difference of the expression $\left(\frac{4}{5}a^2 - ab - 5\right) + \left(\frac{6}{5}a^2 + 2.5ab + 7\right) - (-5a^2 + 6ab + 4)$.

Instructional Item 2

Determine the product of the expression $\left(\frac{1}{2}s + 1\right)\left(\frac{1}{2}s - 1\right)\left(\frac{1}{4}s^2 + 1\right)$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.5

Benchmark

MA.912.AR.1.5 Divide polynomial expressions using long division, synthetic division or algebraic manipulation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1, MA.912.NSO.1.2

Terms from the K-12 Glossary

- Algorithm
- Polynomial

Vertical Alignment

Previous Benchmarks

- MA.7.AR.1.1
- MA.8.AR.1.2, MA.8.AR.1.3
- MA.912.AR.1.4

Next Benchmarks

- MA.912.AR.6.3

Purpose and Instructional Strategies

In Algebra I, students perform operations on polynomials limited to 3 or fewer terms and divide polynomials by a monomial. In Mathematics for College Algebra, students perform operations on all polynomials.

- Instruction includes the connection to addition, subtraction and multiplication of polynomials to develop the understanding of closure, and the connection to properties of exponents.
- Instruction includes proper vocabulary and terminology like dividend, divisor and quotient.
- Instruction includes the connection to long division of whole numbers to develop the understanding that polynomial division follows a very similar procedure and that when the remainder is zero, the divisor and quotient are factors of the dividend *(MTR.5.1)*.

- It is not the expectation of this benchmark that students perform division using a specific method or strategy. Students should be able to compare various methods and use the one most efficient depending on the task.
- Instruction includes the use of manipulatives, like algebra tiles, and various strategies, like the area model and properties of exponents.
  - Area Model
    For example, when dividing $2x^2 + 7x + 6$ by $x + 2$, students can create a table with the same number of rows as the number of terms in the divisor (as shown below).

x		
$+2$		

Then, students can write the first term of the dividend in the upper left rectangle. Have students determine what the length must be if the area of that rectangle (shaded yellow) is $2x^2$.

x	$2x^2$	
$+2$		

Once students determine that its length is $2x^2 \div x$ which is $2x$, they can write the length of the rectangle.

	$2x$	
x	$2x^2$	
$+2$		

Students should be able to see that they can calculate the area of the bottom left rectangle as $2x(2) = 4x$.

	$2x$	
x	$2x^2$	
$+2$	$4x$	

Next, students can use this information to determine the area of the upper right rectangle (shaded green). Students should notice that the second term of the dividend is $7x$, which means that when you add the like terms diagonally you need to get $7x$. The area of this rectangle is $7x - 4x = 3x$.

	$2x$	3
x	$2x^2$	
$+2$	$4x$	

Lastly, students can calculate the length of the column, and then the area of the bottom right rectangle to determine the quotient of $2x^2 + 7x + 6$ and $x + 2$ is $2x + 3$.

	$2x$	3
x	$2x^2$	$3x$
$+2$	$4x$	6

Common Misconceptions or Errors

- Students may forget to write the polynomials in standard form.
- Students may forget to write a zero when there is a missing term either in the divisor or in

the dividend.

- Students may forget that synthetic division can only be used when you are dividing by a linear factor.
- When doing long division, students may add instead of subtracting the terms.
- When doing synthetic division, students may forget that the leading coefficient of the divisor should be 1.
- When doing synthetic division, students may forget to reverse the sign of the constant in the divisor.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.5.1, MTR.7.1)

The volume of a rectangular prism is given by the polynomial $3x^4 + 7x^3 - 12x^2 + 4x$ and the area of the base is given by the polynomial $3x^2 - 2x$.

Part A. Find the height of the rectangular prism.

Part B: Find the length and the width of the rectangular prism.

Instructional Task 2 (MTR.3.1, MTR.4.1)

Part A. Determine the quotient of $4m^4 + 2m^2 - 3m + 2$ and $(2m + 6)$.

Part B. Compare your strategy with a partner.

Instructional Items

Instructional Item 1

Divide $x^4 - 2x^3 + x^2 - 8x - 12$ by $(x + 1)$.

Instructional Item 2

If $(x - 4)$ is a factor of the polynomial $2x^2 - ax - 16$, what is the value of a?

Instructional Item 3

Determine the quotient of $x^4 + 4x^3 - x - 4$ and $x^3 - 1$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.9

Benchmark

MA.912.AR.1.9 Apply previous understanding of rational number operations to add, subtract, multiply and divide rational algebraic expressions.

Benchmark Clarifications:
Clarification 1: Instruction includes the connection to fractions and common denominators.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.2
- MA.912.AR.8.1

Terms from the K-12 Glossary

- Rational expression

Vertical Alignment

Previous Benchmarks

- MA.7.NSO.2
- MA.8.AR.1.2, MA.8.AR.1.3
- MA.912.AR.1.4

Next Benchmarks

- MA.912.AR.8.2, MA.912.AR.8.3

Purpose and Instructional Strategies

In elementary and middle grades students perform operations with rational numbers, including fractions. In Math for College Algebra, they extend the work they did with operations with rational numbers to operations with rational expressions.

- Instruction includes the connection to fractions and common denominators. A rational expression, same as a fraction, can be written as a ratio. Instead of a ratio of two numbers, rational expressions are a ratio of two polynomial expressions.
- The properties of fractions can be applied to rational expressions.
  - Simplifying rational expressions
    Factor the numerator and denominator and extract common factors.
  - Multiply rational expressions
    It may be helpful to first, factor the numerator and the denominator then, multiply the numerators and multiply the denominators.
  - Divide rational expressions
    Students can multiply the first expression by the reciprocal of the second expression and follow the multiplication steps.
  - Adding and subtracting rational expressions
    Students can find a common denominator and then add or subtract the numerators.

Common Misconceptions or Errors

- When simplifying rational expressions students cancel out terms instead of factors. Remind students to always factor before simplifying.
- When simplifying out the entire numerator students forget to write 1 as the numerator.
- When multiplying rational expressions students cross multiply instead of multiplying across.
- When adding and subtracting rational expressions students tend to forget to find a common denominator.

- When adding and subtracting rational expressions students mistakenly follow the steps of multiplication (i.e., add across the denominators and/or add across the numerators).

Instructional Tasks

Instructional Task 1(MTR.3.1, MTR.5.1, MTR.7.1)

The U.S. Department of Energy keeps track of fuel efficiency for all vehicles sold in the United States. Each car has two fuel economy numbers, one measuring efficient for city driving and one for highway driving. For example, a 2012 Volkswagen Jetta gets 29.0 miles per gallon (mpg) in the city and 39.0 mpg on the highway.

Many banks have "green car loans" where the interest rate is lowered for loans on cars with high combined fuel economy. This number is not the average of the city and highway economy values. Rather, the combined fuel economy (as defined by the federal Corporate Average Fuel Economy standard) for x mpg in the city and y mpg on the highway, is computed as $\frac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)}$.

Part A. What is the combined fuel economy for the 2012 Volkswagen Jetta? Give your answer to three significant digits.

Part B. For most conventional cars, the highway fuel economy is 10 mpg higher than the city fuel economy. If we set the city fuel economy to be x mpg for such a car, what is the combined fuel economy in terms of x? Write your answer as a single rational function $\frac{a(x)}{b(x)}$.

Part C. Rewrite your answer from Part B in the form of $q(x)+\frac{r(x)}{b(x)}$ where and $q(x)$, $r(x)$ and $b(x)$ are polynomials and the degree of $r(x)$ is less than the degree of $b(x)$.

Part D. Use your answer in Part C to conclude that if the city fuel economy, x, is large, then the combined fuel economy is approximately $x+5$.

Instructional Task 2 (MTR.5.1, MTR.7.1)

Tamisha walked 4 miles to the store to buy a new bike at an average speed of x miles per hours. She returned home riding her bike and her average speed was 2 miles faster than walking.

Part A. Write an expression for the time Tamisha takes to get to the store.
Part B. Write an expression for the time Tamisha takes to get home from the store.
Part C. Write an expression for the total time she spent on both trips.
Part D. If Tamisha spent 3 hours on both trips, what is Tamisha's average walking speed.

Instructional Items

Instructional Item 1 (MTR.3.1)

Perform the operation(s) on the rational expression. Write your answer as single rational expression.

a. $\frac{x^2-5x-24}{6x+2x^2} \cdot \frac{5x^2}{8-x}$

b. $\frac{9-x^2}{x^2+5x+6} \div \frac{2x-6}{5x+10}$

c. $\frac{5}{3x} + \frac{2}{7x} - \frac{1}{2x}$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2 *Write, solve and graph linear equations, functions and inequalities in one and two variables.*

MA.912.AR.2.4

Benchmark

MA.912.AR.2.4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain, range, intercepts and rate of change.
Clarification 2: Instruction includes the use of standard form, slope-intercept form and point-slope form.
Clarification 3: Instruction includes cases where one variable has a coefficient of zero.
Clarification 4: Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.
Clarification 5: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder notations.

Connecting Benchmarks/Horizontal Alignment

- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function notation
- Range
- Rate of change
- Set-builder notation
- Slope
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks **Next Benchmarks**

- MA.8.AR.3.4
- MA.912.AR.2.2
- MA.912.NSO.4.2
- MA.912.AR.9.8, MA.912.AR.9.9, MA.912.AR.9.10

Purpose and Instructional Strategies

In grade 8, students graphed two-variable linear equations given a written description, a table, or an equation in slope-intercept form. In Algebra I, students graphed linear functions and determined and interpreted key features. In Math for College Algebra, students graph linear functions focusing on real-world contexts.

- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y | y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation
      If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Within this benchmark, linear two-variable equations include horizontal and vertical lines. Instruction includes writing horizontal and vertical lines in the form $y = 3$ and $x = -4$ and as $0x + 1y = 3$ and $1x + 0y = -4$, respectively. Students should understand that vertical lines are not linear functions, but rather linear two-variable equations.
    - Discussions about this topic are a good opportunity to foreshadow the use of horizontal and vertical lines as common constraints in systems of equations or inequalities.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.

Common Misconceptions or Errors

- Students may express initial confusion with the meaning of (x) for functions written in function notation. To help address this, consider writing the same function in both forms simultaneously *(MTR.2.1)*, i.e., $f(x) = \frac{2}{3}x + 6$ and $y = \frac{2}{3}x + 6$, to show that both $f(x)$ and y represent the same outputs of the function.
- Students may misunderstand representing interval notation from smallest to largest when representing the domain or range. To address this misconception, have students sketch the graph to demonstrate their understanding of representing the domain from left to right and range from bottom to top of their graph.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.7.1)

There are a total of 549 seniors graduating this year. The seniors walk across the stage at a rate of 42 seniors every 30 minutes. The ceremony also includes speaking and music that lasts a total of 25 minutes.

Part A. Write a function that models this situation. Compare your function with a partner.

Part B. Graph the function you created in Part A. What is the feasible domain and range for this situation?

Part C. For how many hours will the graduation ceremony take place?

Instructional Task 2 (MTR.3.1, MTR.7.1)

The table shows the amount of money, D, in a savings account after m months.

Number of Months (m)	2	5	9	11
Dollar Amount (D)	740	1100	1580	1820

Part A. Write a function that models this situation.

Part B. What is the y-intercept of the function you created? How does it mean in this situation?

Part C. Graph the function you created in Part A. What is the feasible domain and range for this situation?

Part D. How many months does it take to have \$3,260 in the saving account?

Instructional Task 3 (MTR.3.1, MTR.4.1, MTR.7.1)

Martha spent \$109 renting chairs and tables for her party. Each table costs \$9.80 and the chairs \$2.00 each. The equation $2c + 9.8t = 129$ represents the relationship between the chairs, c, the tables, t, and the dollar amount Martha spent renting these items.

Part A. Graph the equation that represents the situation described above. What does the point $(5, 40)$ mean in this situation?

Part B. What is the feasible domain and range for this situation?

Part C. Identify the x- and y-intercepts. What do they mean in this situation? Explain your reasoning.

Instructional Items

Instructional Item 1

Part A. Graph the function $g(x) = -3.6x + 7$.

Part B. Identify the domain, range and any intercepts of the function.

Instructional Item 2

Identify the slope and the y-intercept of the linear function.

x	-3	1	7	9	18
$f(x)$	-2.62	-1.66	-0.22	0.26	2.42

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2.5

Benchmark

MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.

Algebra I Example: Lizzy's mother uses the function $C(p) = 450 + 7.75p$, where $C(p)$ represents the total cost of a rental space and p is the number of people attending, to help budget Lizzy's 16th birthday party. Lizzy's mom wants to spend no more than \$850 for the party. Graph the function in terms of the context.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain, range, intercepts and rate of change.
Clarification 2: Instruction includes the use of standard form, slope-intercept form and point-slope form.
Clarification 3: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.
Clarification 4: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.
Clarification 5: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.2
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.9.6, MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function notation
- Range
- Rate of change
- Slope
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.4, MA.8.AR.3.5
- MA.912.F.1.5

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students determined and interpreted the slope and y-intercept of a two-variable linear equation in slope-intercept form from a real-world context. In Algebra I, students solved real-world problems modeled with linear functions. In Mathematics for College Algebra, students will graph and solve problems involving linear functions.

- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-builder notation
    If the range is all values of y less than or equal to zero, it can be represented as

$\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation
      If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
    - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
    - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
    - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
    - For example, if the range of a function is all real numbers, is can be represented in interval notation as $(-\infty, \infty)$. This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Instruction includes the use of x-y notation and function notation for all forms of linear functions (slope-intercept, point-slope and standard).
- Instruction includes making the connection to constraints (MA.912.AR.9.6) for a given context. Students should develop an understanding that linear graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by linear functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
- Instruction includes the understanding that a real-world context can be represented by a linear two-variable equation even though it only has meaning for discrete values.
    - For example, if a gym membership cost $\$10.00$ plus $\$6.00$ for each class, this can be represented as $y = 10 + 6c$. When represented on the coordinate plane, the relationship is graphed using the points $(0,10), (1,16), (2,22)$, and so on.
- For mastery of this benchmark, students should be given flexibility to represent real-world contexts with discrete values as a line or as a set of points.
- Instruction directs students to graph or interpret a representation of a context that necessitates a constraint. Discuss the meaning of multiple points on the line and announce their meanings in the associated context *(MTR.4.1)*. Allow students to discover that some points do not make sense in context and therefore should not be included in a formal solution *(MTR.6.1)*. Ask students to determine which parts of the line create sensible solutions and guide them to make constraints to represent these sections.
- Instruction includes the use of technology to interpret key features and develop the understanding of constraints.

Common Misconceptions or Errors

- Students may assign their constraints to the incorrect variable.

- Students may miss the need for compound inequalities in their constraints. Students may not include zero as part of the domain or range.
    - For example, if a constraint for the domain is between 0 and 10, a student may forget to include 0 in some contexts, since they may assume that one cannot have negative people, for instance.
- Students may confuse an open interval with an ordered pair.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.3.1)

Sharisse is enrolling her daughter in daycare that costs $75.00 per day and there is an initial fee to reserve the spot of $300.

Part A. Taking into account weekends, write a linear function that could describe this situation.

Part B. Sharisse is planning for her monthly budget. If she only has her daughter attend daycare 3 days a week. How much would Sharisse have to pay?

Part C. Share with a partner how you determine the amount for Part B.

Instructional Task 2 (MTR.7.1)

A coffee truck has a high volume urn that makes 3 gallons of coffee. The coffee truck sets up in a local park at 7:00 am and sells 12 ounces cups for $2.50.

Part A. Create a graph describing the relationship of how much coffee is left in the urn and how many cups of coffee are sold. Is there a linear relationship between the amount of coffee in the urn and how many cups of coffee are sold?

Part B. What would be the domain and range for the graph created in Part A?

Instructional Items

Instructional Item 1

Use the function $g(x) = 6.25 - 15x$ to answer the following.

Part A. Determine its intercepts.

Part B. Write a situation that could be described by this function.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3 *Write, solve and graph quadratic equations, functions and inequalities in one and two variables.*

MA.912.AR.3.7

Benchmark

MA.912.AR.3.7 Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.
Clarification 2: Instruction includes the use of standard form, factored form and vertex form, and sketching a graph using the zeros and vertex.
Clarification 3: Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.
Clarification 4: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.8
- MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function notation
- Quadratic function
- Range
- Set builder notation
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.3.1, MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students were introduced to quadratic functions and learned to wrote, graph and interpret their key features. In this course, students build on the foundation to focus on quadratic functions within real-world contexts.

- Instruction includes conversations about interpreting y-intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and symmetry.
- When discussing end behavior, students should see a relationship between the sign of a and the end behavior the function exhibits. Instruction presents students with the equation of the function first, before showing its graph and asking them to predict its end behavior *(MTR 5.1)*.
  - Depending on the form the function is presented in, students may be able to predict other features as well *(MTR 5.1)*.
- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when one form might be

more useful than other depending on the context.

- Standard Form
  Can be described by the equation $y = ax^2 + bx + c$, where a, b and c are any rational number. This form can be useful when identifying the y-intercept.
- Factored form
  Can be described by the equation $y = (x - r_1)(x - r_2)$, where r_1 and r_2 are real numbers and the roots, or x-intercepts. This form can be useful when identifying the x-intercepts, or roots.
- Vertex form
  Can be described by the equation $y = a(x - h)^2 + k$, where the point (h, k) is the vertex. This form can be useful when identifying the vertex.

- Instruction includes the use of x-y notation and function notation.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation.
  - Words
    If the domain is all real numbers, it can be written as “all real numbers” or “any value of x, such that x is a real number.”
  - Inequality notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-builder notation
    If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as “all values of y such that y is less than or equal to zero.”
  - Interval notation
    If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.
- Instruction includes opportunities for students to compare functions and show similarities between key features. It is also important that students see that the whole graph is symmetric across the vertex and if one point is found on one half of the graph, we can use its twin on the other half.

Common Misconceptions or Errors

- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.
- When using interval notation, students may confuse when to use the parentheses and the brackets appropriately. In these cases, ask reflective questions that will encourage the students to connect their knowledge about when to use the greater than or equal to/less

than or equal to or just greater than/less than as well as the closed circle and open circles on the number line.

- When using interval notation, students may confuse when to use the x-values (domain/increasing, decreasing, constant, etc.) and when to use the y-values (range). In these cases, encourage the students to connect the graphical representation to their interval notation.

Instructional Tasks

Instructional Task 1 (MTR 2.1, MTR 4.1)

A given function is represented by the table below.

x	$f(x)$
0	-1
0.5	0.25
1	1
1.5	1.25
2	1
2.5	0.25

Part A. Graph the function.
Part B. Determine the y-intercept and describe how you found it.
Part C. Over what interval is the function increasing? Over what interval is it decreasing?
Part D. What is the equation for the axis of symmetry? Describe how you found the axis of symmetry.
Part E. What is the domain of the function? What is the range of the function?
Part F. Describe which representation, table or graph, is easier to use to find the key features of this function.

Instructional Items

Instructional Item 1

Graph the function $f(x) = -2(x + 6)(x + 3)$. Identify the domain, range, vertex, and zeros of the function.

Instructional Item 2

Graph the function represented in the table below. Describe the domain, range, vertex, and end behavior of the function.

x	$f(x)$
0	7.5
.5	11.25
1	12.5
1.5	11.25
2	7.5
2.5	1.25

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3.8

Benchmark

MA.912.AR.3.8 Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.

Algebra I Example: The value of a classic car produced in 1972 can be modeled by the function $V(t) = 19.25t^2 - 440t + 3500$, where t is the number of years since 1972. In what year does the car's value start to increase?

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.

Clarification 2: Instruction includes the use of standard form, factored form and vertex form.

Clarification 3: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Clarification 4: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.2
- MA.912.AR.2.5
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.9.6, MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function Notation
- Quadratic Function
- Range
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.3.1, MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students were introduced to quadratic functions and learned to write, graph and interpret their key features. In this course, students build on the foundation to focus on quadratic functions within real-world contexts.

- Instructional expectation of this benchmark should reinforce core concepts and extend student knowledge through problem solving and real-world applications.
- Instruction features a variety of real-world contexts. Some of these contexts should require students to create a function as a tool to determine requested information or should provide the graph or function that models the context.
- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when one form might be more useful than other depending on the context.
    - Standard Form
      Can be described by the equation $y = ax^2 + bx + c$, where a, b and c are any rational number. This form can be useful when identifying the y-intercept.
    - Factored form

Can be described by the equation $y = (x - r_1)(x - r_2)$, where r_1 and r_2 are real numbers and the roots, or x-intercepts. This form can be useful when identifying the x-intercepts, or roots.
  - Vertex form
    Can be described by the equation $y = a(x - h)^2 + k$, where the point (h, k) is the vertex. This form can be useful when identifying the vertex.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation, and interval notation.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-builder notation
    If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
  - Interval notation
    If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
  - For example, if the range of a function is all real numbers, is can be represented in interval notation as $(-\infty, \infty)$. This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.

Common Misconceptions or Errors

- Students may find themselves stuck initially, unsure of where to start. In conversations with these students, prompt them to reflect on what they know about the context and how they can use that information to determine the requested information *(MTR.1.1)*.

  - For example, students may have an equation in standard form and need to interpret the vertex in context. Prompting students to consider how they've calculated vertices in the past should lead them to choose to either convert the equation into vertex form or use the line of symmetry to help determine the vertex.

- Students misunderstanding of recognizing the vertex form is most useful for determining max or min may hinder some students from appropriately connecting concepts in problems.
  - For example, students may not understand the meaning to the vertex and will solve the original quadratic function by setting it equal to 0.
- Students may interpret $f(x) = 0$ as equivalent to $f(0)$.
- Students may struggle interpreting key features and graphing from various forms of quadratic functions.
  - For example, students may be asked to find the vertex and may believe that completing the square is the only form to use. Prompting students to consider different forms will help with understanding the meaning of key features.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.5.1)

ABC Pool Company is constructing a 17 feet by 11 feet rectangular pool. Along each side of the pool, they plan to construct a concrete sidewalk that has a constant width. The land parcel being used has a total area of 315 sq. ft. to construct the pool and sidewalks. The function $f(x) = 4x^2 + 56x - 128$ represents the situation described. Transform the function to determine and interpret its zeros.

Instructional Task 2 (MTR.4.1)

During a math test, Charlie is given the equation $4x^2 - 9x + 18 = 0$ and asked to find its discriminant.

Part A. What is the discriminant of the equation? What does this discriminant tell Charlie about the nature of the solutions to the equation?

Part B. Once he finds the discriminant, Charlie is asked to solve the equation. What are the solutions to the equation?

Part C. After solving this equation, Charlie is given a new quadratic equation with real coefficients and is asked to solve it. He determines that there are two solutions, one real root and one imaginary root. Are Charlie's solutions, correct? Please explain your answer.

Instructional Task 3 (MTR.5.1)

Part A. Determine the values of x that satisfy the equation $-x^2 + 3x - 1 = 0$. If exact roots cannot be found, state the consecutive integers between which the roots are located.

Part B. Use the equation of the axis of symmetry to sketch the graph of the function $-x^2 + 3x - 1 = y$.

Part C. Create a table to help identify the x-intercepts of the of the function $-x^2 + 3x - 1 = y$.

Part D. Identify possible solutions to the of the function $-x^2 + 3x - 1 = y$.

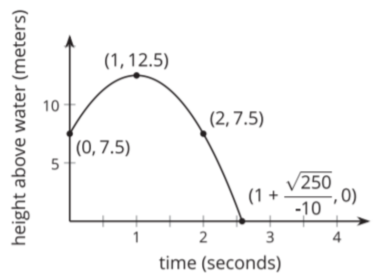
Instructional Items

Instructional Item 1

The Chrysler Building, which opened in 1930, is one of New York's oldest buildings with a roof height of 925 feet (282 meters). Suppose you could conduct an experiment by dropping a small object from the roof of the Chrysler Building. Using the formula $h(t) = -16t^{2} = h_0$, where t is the time in seconds and the initial height h_0 in feet, how long would it take for the object to reach the ground, assuming there is no air resistance?

Instructional Item 2

The function, h, defined by $h(t) = -5t^2 + 10t + 7.5$, models the height of a diver above the water (in meters), seconds after the diver leaves the board.



Part A. How high above the water is the diving board?
Part B. When does the diver hit the water?
Part C. What is the maximum height the diver reaches during the dive?
Part D. After how long will the diver reach the same height as when he started?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.4 *Write, solve and graph absolute value equations, functions and inequalities in one and two variables.*

MA.912.AR.4.2

Benchmark

MA.912.AR.4.2 Given a mathematical or real-world context, write and solve one-variable absolute value inequalities. Represent solutions algebraically or graphically.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.9.10

Terms from the K-12 Glossary

- Absolute value
- Compound inequality

Vertical Alignment

Previous Benchmarks

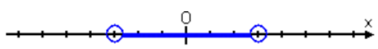
- MA.912.AR.4.1

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students solved one-variable absolute value equations. In Math for College Algebra, students write and solve one-variable absolute value inequalities.

- Instruction includes making the connection between solving absolute value equations to solving absolute value inequalities. Where the solution to an absolute value equations is points, the solution to an absolute value inequality is going to be intervals.
- Instruction includes using words to help determine solutions.
  - For example, the equation $|x| < 3$ can be read as "Which values of x are less than 3 units away from zero?" This can be represented algebraically as $-3 < x < 3$ and the graphically solution looks like the number line below.



- Instruction includes the connection to compound inequalities.
  - When solving the inequality $|x - 1| \leq 3$, students may set up as $x - 1 \leq 3$ and $x - 1 \geq -3$.
  - When solving the inequality $|x - 1| > 3$, students may set up as $x - 1 > 3$ or $x - 1 < -3$.
- Instruction encourages students to discuss their thinking with their peers *(MTR.4.1)*. Instructional focus should include conversations about the solution set algebraically and graphically. Often students will be able to reason out their thinking, but will struggle with representing their thinking mathematically. Encourage this process and ask questions that will help with solving the task *(MTR.1.1)*. Are there multiple ways for students to represent this problem mathematically *(MTR.2.1)*?
- Instruction includes student understanding that compound inequalities with "or" create a combining (or a union) of the solutions of the individual inequalities while compound inequalities with "and" create an overlap (or an intersection) of the solutions of the individual inequalities.

Common Misconceptions or Errors

- Students may confuse the inequality symbols.
- Students may forget that multiplication or division by a negative number will change the direction of the inequality symbol.
- Students may incorrectly identify the solutions graphically.
- Students may not create two inequalities from an absolute value inequality.
- Given that students' prior experience has dealt with equation work far more than inequality work, students will likely forget to consider the direction of the inequality symbol as they solve inequalities. To address this, have these students consider a context to understand the need to pay attention to the direction of the inequality symbol.
- Students may represent the absolute value inequality using the incorrect compound inequality. It may be helpful to have students develop a graphic organizer to help organize steps.
- In interval notation, students may misinterpret the symbol for the union or intersection of two sets and indicate values incorrectly as being included as part of the solution set.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Ms. Pruitt's candy company sells boxes of their famous chocolate-covered peanut butter bites. The candy is sold in boxes that must be within 95 grams of weighing 1,465 grams. Each piece weighs 42.5 grams and the box weighs 116 grams. Calculate and graph the possible quantities of chocolate-covered peanut butter bites that could be in a single box.

Instructional Task 2 (MTR.6.1, MTR.7.1)

Great Gardens Landscaping has been hired to build gardens around a new resort. They designed plans for two different rectangular gardens. In the first plan the length is twice the length of the width and in the second plan the length is 2 feet longer than the width. The perimeters of the gardens cannot differ more than 3 feet.

Part A. Write and solve an inequality to determine the range of the widths of the garden.

Part B. What are the dimensions of the two gardens using the smallest allowable width?

Garden 1: Garden 2:

Part C. Using the largest allowable width, what is the area of the largest garden?

Instructional Task 3 (MTR.6.1, MTR.7.1)

Tom is a machinist at Ancient City Choppers. To ensure the highest reliability of the motorcycle engines produced at the shop, Tom machines each cylinder to a 4.01' diameter with a tolerance of 0.002".

Part A. Write an absolute value inequality that could be used to determine the acceptable range of each cylinder's diameter.

Part B. Express that range graphically.

Instructional Task 4

The National Coffee Association says brewers should have an optimal brewing temperature of 200 degrees Fahrenheit, plus or minus five degrees.

Part A. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.

Part B. If the standard temperature in a Keurig is 192 degrees Fahrenheit, would you recommend that this product is meeting the National Coffee Association recommendations for an excellent cup of coffee? Explain.

Instructional Items

Instructional Item 1

There is a cloud layer centered at 4 miles above the sea that is 1,000 feet thick. At what elevations, x, given in miles, will a plane in that area be out of the clouds?

Instructional Item 2

Determine the solutions to the absolute value inequality below.

$$|2x + 3| \leq 6$$

Instructional Item 3

The manufacturing plant for a popular sugar brand fills each of its bags with 16 ounces of sugar to be sent to local grocery stores. After the bags are filled, another machine weighs them to ensure accuracy. If the bag weighs 0.7 ounces more or less than the desired weight, the bag is rejected.

Write an equation to find the heaviest and lightest bag the machine will approve. Include a verbal and graphical explanation of possible values that are acceptable.

***The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.4.4

Benchmark

MA.912.AR.4.4	Solve and graph mathematical and real-world problems that are modeled with absolute value functions. Interpret key features and determine constraints in terms of the context.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; vertex; end behavior and symmetry.

Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Absolute value
- Coordinate plane

Vertical Alignment

Previous Benchmarks **Next Benchmarks**

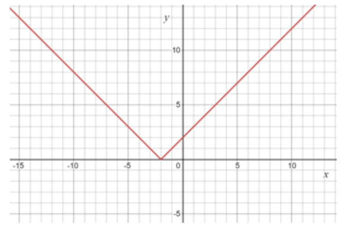
- MA.912.AR.4.3

Purpose and Instructional Strategies

In Algebra I, students graphed absolute value functions and interpreted its key features. In Math for College Algebra, students solve problems modeled with absolute value functions, determining key features and constraints within context.

- When making connections to transformations of functions, use graphing software to explore absolute value functions adding variability to the parent equation to see the effects on the graph. Allow students to make predictions *(MTR.4.1)*.
- Instruction includes making the connection between the algebraic and graphical representations. In doing so, students can tie together key feature concepts of absolute value functions and provides opportunities to make connections to linear functions and its key features.
- Students should understand that there are two ways in which absolute value functions can be represented algebraically *(MTR.5.1)*.
  - Using absolute value notation $f(x) = |x|$.
  - Using piecewise notation $f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$.
    - The piecewise graph focuses on what should happen algebraically to the input value that will give its distance from the origin.
- Instruction includes utilizing students' knowledge of transformations to graph absolute value functions that are not parent functions *(MTR.2.1)*.
  - For example, given $f(x) = |x + 3|$, to graph as a piecewise students can set the expression inside of the absolute value equal to zero to solve for the x-value where the function equals zero. So any value of x less than -3 and any value of x greater than -3.
    - Using Piecewise Notation $f(x) = \begin{cases} x + 3, & if\ x \geq -3 \\ -x - 3, & if\ x < 3 \end{cases}$
    - Using the transformation criteria, students should understand that the graph of this function is a horizontal shift to the left. Points can be plotted using the slope of the function and other key features that it provides.
- Instruction reinforces the definition of absolute value as a number's distance from zero (0) on a number line. For absolute value functions, noticing where the argument of the absolute value bars will be zero can be helpful in ensuring that students graph correctly.
  - For example, if students are asked to graph $y = |x + 2|$, students should recognize that the absolute value will be a V-shaped graph and the sharp bend will occur at the value of x that makes the math statement of the function, $|x + 2|$, equal to zero. Notice in the table below the value that makes the math statement zero is -2 and to graph using a table, students can choose x values to the left and right of that critical value, -2.

x	$\|x+2\|$
-4	2
-3	1
-2	0
-1	1
0	2



- Instruction allows for the flexibility to graph absolute value functions using a variety of ways.
    - For example, the function $y = |x + 2|$ can be graphed using transformations, recognizing that this graph shifts two units to the left and will open upward because there's a positive a value.
    - For example, the function $y = |x + 2|$ can be graphed by plotting points from the vertex, using the "slope." In this case, once the vertex is plotted, as you travel 1 unit to the left and right you would go up one unit.
    - For example, for the function $y = |x + 2|$, students can create a T-chart and solve for the values that satisfy the function. It is important that students include negative inputs in their T-chart so they are not misled as to what the graph looks like. Students can use their understanding of the parent function of $y = |x|$, whose v-shaped graph has a center of 0. In creating and plotting points, students will use the vertex to find the center of the graph and then choose additional point x-values to use in the table to the left and right of the vertex.
- Instruction includes representing domain, range and constraints using words, inequality notation, set-builder notation and interval notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality Notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-Builder Notation
      If the domain is all values of x less than or equal to zero, it can be represented as $\{x|x \leq 0\}$ and is read as "all values of x such that x is less than or equal to zero."
    - Interval Notation
      If the domain is all values from 0 to infinity, including 0, it can be represented as $[0, \infty)$.
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
    - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
    - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.

- For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
- For example, if the range of a function is all real numbers, is can be represented in interval notation as $(-\infty, \infty)$. This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).

- Instruction includes the use of x-y notation and function notation for absolute value functions.
- Instruction includes making the connection to constraints (MA.912.AR.9.6) for a given context. Students should develop an understanding that absolute value graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by absolute value functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.
- Depending on a student's pathway in the future, it will be important to understand that there is a relationship between piecewise functions and absolute value functions when working with derivatives or integration of functions *(MTR.5.1)*.
- When addressing real-world contexts, the absolute value is used to define the difference or change from one point to another. Connect the graph of the function to the real-world context so the graph can serve as a model to represent the solution *(MTR.6.1, MTR.7.1)*.
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.
- Problem types include applying distance to real-world problems, showing how much a value deviates from the norm.
- Instruction includes opportunities for students to compare functions and show similarities between key features. It is also important that students see that the whole graph is symmetric across the vertex and if one point is found on one half of the graph, we can use its twin on the other half.

Common Misconceptions or Errors

- Students may think that distributive property should be used if a constant is in front of the absolute value.
  - For example, for $f(x) = 4|x - 2|$, students will incorrectly distribute the 4 to the absolute value function when evaluating.
- Students may not fully understand the connection of all of the key features (emphasize the use of technology to help with student discovery) and how to represent them using the proper notation.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

Minivans, due to their size and purpose, are not typically very fuel-efficient vehicles. The EPA's estimates for fuel efficiency for the Toyota 2020 Sienna AWD is 14 city miles.

Part A. Write a function that describes the Sienna's estimated city miles per gallon fuel efficiency.

Part B. Compare your function with a partner.

Part C. Describe the graph of the function from Part A.

Instructional Task 2 (MTR.3.1, MTR.5.1)

Part A. Graph $g(x) = -|x-4| + 2$.

Part B. Compare the graph from Part A to its parent function $f(x) = |x|$.

Part C. Use the information from Part A and B to determine the domain and range of the function $g(x)$.

Instructional Items

Instructional Item 1

Electrical parts, such as resistors and capacitors, come with specified values of their operating parameters: resistance, capacitance, etc. However, due to imprecision in manufacturing, the actual values of these parameters vary somewhat from piece to piece, even when they are supposed to be the same. The best that manufacturers can do is to try to guarantee that the variations will stay within a specified range, often $\pm 1\%, \pm 5\%$ or $\pm 10\%$. Suppose we have a resistor rated at 680 ohms, $\pm 5\%$. Use an absolute value function to express the range of possible values of the actual resistance.

Instructional Item 2

Graph the function $f(x) = -0.2|x+7|$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5 *Write, solve and graph exponential and logarithmic equations and functions in one and two variable.*

MA.912.AR.5.2

Benchmark

MA.912.AR.5.2	Solve one-variable equations involving logarithms or exponential expressions. Interpret solutions as viable in terms of the context and identify any extraneous solutions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.7.1
- MA.912.AR.8.1

Terms from the K-12 Glossary

- Base of an exponent
- Exponent (exponential form)
- Exponential function

Vertical Alignment

Previous Benchmarks

- MA.912.F.1.8
- MA.912.FL.3.4

Next Benchmarks

- MA.912.DP.2.9
- MA.912.C.2.4
- MA.912.C.2.8

Purpose and Instructional Strategies

In Algebra I, students worked with the properties of exponents and solved one-variable linear, quadratic and exponential equations. In Math for College Algebra, students will learn that there are similar rules for logarithms and when some exponential equations can't be solved, one will use logarithms to help convert the bases. In Math for College Algebra, students are building on their skills to solve equations involving exponents or logarithms. In later courses, students will use derivatives to solve different types of function problems.

- Instruction includes uses various methods to solve one-variable exponential equations.
  - For example, students can use the properties of exponents or logarithms to determine the solution for the equation $2^x = 32$.
    - Students can rewrite the equation as $2^x = 2^5$, noticing that since the base of both sides of equations are the same, then the exponents must be equivalent to one another. Therefore, $x = 5$.
    - Students can rewrite the equation as $\log_2 32 = x$, noticing that they can use the change of base formula to determine the value of x. Therefore, $x = \frac{\log 32}{\log 2}$, which is equal to 5.
- Instruction includes solving logarithmic equations by changing from a logarithmic expression to an exponential expression.
  - For example, when solving $\log_3(4x - 7) = 2$, we can obtain an exact solution by changing the logarithm to exponential form creating the equivalent equation $4x - 7 = 3^2$. Students should recognize that this is a one-variable linear equation $4\text{x} - 7 = 9$, and therefore, $x = 4$.

Common Misconceptions or Errors

- Students may not understand the relationship of logarithm and exponential functions and may treat them as separate concepts.
- Students may forget to check for extraneous solutions.
  - For example, when asked to solve $\log_x 36 = 2$, students can solve using the square method and get ± 6. The base of the logarithm is always positive, so we discard the -6.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Between 7:00 a.m. and 9:00 a.m. cars arrive at Starbucks drive-thru at the rate of 24 cars per hour (0.40 car per minute). The following function can be used to determine the probability that a car will arrive within t minutes of 7:00 a.m.

$$C(t) = 1 - e^{-0.2t}$$

Part A. Determine the probability that a car will arrive within 5 minutes of 7 a.m. (that is, before 7:05 a.m.).

Part B. Determine the probability that a car will arrive within 30 minutes of 7 a.m.

Part C. What does the value $C(t)$ approach as t becomes unbounded in the positive direction?

Part D. Graph $C(t)$ using graphing technology.

Instructional Items

Instructional Item 1

Determine the value of x that satisfies the equation $3^{4x-1} = \frac{1}{27}$.

Instructional Item 2

Solve for x in the equation $2^x = 11$.

Instructional Item 3

Solve for x in the equation $log_6(2x - 3) = log_6(x + 2)$.

Instructional Item 4

Determine the value of x that satisfies the equation $2log_{10}6 - \frac{1}{3}log_{10}27 = log_{10}x$.

Instructional Item 5

Solve for x in the equation $log_2(x) = 3$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.4

Benchmark

MA.912.AR.5.4	Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.

Benchmark Clarifications:

Clarification 1: Within the Algebra I course, exponential functions are limited to the forms $f(x) = ab^x$, where b is a whole number greater than 1 or a unit fraction, or $f(x) = a(1 \pm r)^x$, where $0 < r < 1$.

Clarification 2: Within the Algebra I course, tables are limited to having successive nonnegative integer inputs so that the function may be determined by finding ratios between successive outputs.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1
- MA.912.F.1.1, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Exponential function
- Exponents
- Function

Vertical Alignment

Previous Benchmarks

- MA.912.AR.5.3

Next Benchmarks

- MA.912.AR.5.5, MA.912.AR.5.7
- MA.912.AR.8.2

Purpose and Instructional Strategies

In Algebra I, students wrote exponential functions that modeled relationships characterized by having a constant percent of change per unit interval. In Math for College Algebra, students continue this work, not limited to the forms $f(x) = ab^x$, where b is a whole number greater than 1 or a unit fraction, or $f(x) = a(1 \pm r)^x$, where $0 < r < 1$. Additionally, students further develop their understanding of this feature of exponential functions from Algebra I by emphasizing real-world context and applications.

- Instruction includes guidance on how to determine the initial value or the percent rate of change of an exponential function when it is not provided.
  - For example, if the initial value of $(0,3)$ is given, students can now write the function as $f(x) = 3b^x$. Guide students to choose a point on the curve that has integer coordinates such as $(2,12)$. Lead them to substitute the point into their function to find b. Students should recognize that exponential functions are restricted to positive values of b, leading to the function $f(x) = 3(2)^x$.
- Instruction includes interpreting percentages of growth and decay from exponential functions.
  - For example, the function $(x) = 500(1.16)^x$ represents 16% growth of an initial value.
    - Guide students to discuss the meaning of the number 1.16 as a percent. They should understand it represents 116%. Taking 116% of an initial value increases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential growth.
  - For example, the function $(x) = 500(0.72)^x$ represents 28% decay of an initial value.
    - Guide students to discuss the meaning of the number 0.72 as a percent. They should understand it represents 72%. Taking 72% of an initial value decreases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential decay.
  - For example, the function $(x) = 500(1)^x$ represents an initial value that neither grows nor decays as x increases.
    - Guide students to discuss the meaning of the number 1 when it comes to growth/decay factors. They should understand it represents 100%. Taking 100% of an initial value causes the value to remain the same. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to no change in the initial value (explaining the horizontal line that shows when $b = 1$ on the graph).
- Use prior knowledge on transformations of functions to explore different forms of exponential functions.
  - For example, the function $f(x) = 16\left(\frac{1}{2}\right)^x$ can be written as $f(x) = 4\left(\frac{1}{2}\right)^{x-2}$ from the given table of values, showing the horizontal shift.

$\boldsymbol{x}$	0	1	2	3
$\boldsymbol{f(x)}$	16	8	4	2

- Instruction includes students using an irrational number, symbolized by the letter e, as the base in many applied exponential functions. The number e is called the natural base and can be approximated to 2.72. It is used when the relationship between two quantities is growing exponentially at a continuous rate.

Common Misconceptions or Errors

- Students may not recognize that if b is greater than 1 that it represents growth and if b is less than one that it represents decay.
- Students may struggle differentiating between a constant rate of change and common ratio to determine which type of function they are writing.
- Students may confuse written form of decimals and percentages, e.g., 5% growth $(1 + .05) = 105\ \%$ not equal to 150%.
- Remind students to use multiple points when given a table to help find the exponential function.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1, MTR.7.1)

Raylene and Caleb were working on separate biology experiments. Each student documented their cell counts over time in the chart below.

Raylene's Experiment	
# of minutes	*# of cells*
3	2400
6	1200
9	600
12	300
15	150

Caleb's Experiment	
# of minutes	*# of cells*
0	160
1	240
2	360
3	540
4	910

Part A. Do the number of cells in Caleb's experiment increase at a constant percentage rate of change? If so, what is the percentage rate? If not, describe what is happening to the number of cells. Does this change represent growth or decay? Justify your answer.

Part B. Write exponential functions to represent the relationship between the quantities for each student's experiment. In which experiment are the number of cells changing more rapidly? Justify your answer.

Part C. Graph these functions and determine their key features.

Instructional Task 2 (MTR.2.1, MTR.7.1)

The population of Zora City in 2019 was estimated to be 83,600 people with an annual rate of increase of 1.8%.

Part A. Write an equation to model future growth.

Part B. What is the growth factor for Zora City?

Part C. Use the equation to estimate the population in 2083 to the nearest hundred people.

Instructional Items

Instructional Item 1

A forester has determined that the number of fir trees in a forest is decreasing by 3% per year. In 2010, there were 13,000 fir trees in the forest. Write an equation that represents the number of fir trees, N, in terms of t, the number of years since 2010.

Instructional Item 2

Write an exponential function for the table below.

x	0	1	2	3	4	5
y	1	4	16	64	256	1024

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.6

Benchmark

MA.912.AR.5.6 Given a table, equation or written description of an exponential function, graph that function and determine its key features.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior and asymptotes.
Clarification 2: Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.
Clarification 3: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.
Clarification 4: Within the Algebra I course, exponential functions are limited to the forms $f(x) = ab^x$, where b is a whole number greater than 1 or a unit fraction, or $f(x) = a(1 \pm r)^x$, where $0 < r < 1$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.2
- MA.912.AR.2.5
- MA.912.AR.3.7
- MA.912.AR.4.4
- MA.912.F.1.1, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function
- Function notation
- Range
- Set builder notation
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA. 912.AR.5.3, MA.912.AR.5.5

Next Benchmarks

- MA.912.AR.7.2
- MA.912.AR.8.2
- MA.912.AR.9.6

Purpose and Instructional Strategies

In Algebra I, students graphed exponential functions and determined their key features, including asymptotes and end behavior. In Math for College Liberal Arts, students continue this work, without the limitations on function forms given in Algebra I.

- Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will discover asymptotes are useful guides to complete the graph of a function, especially when drawing them by hand. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.
    - Instruction should not be limited to exponential functions of the form $\mathrm{f}(x) = ab^x$ as this was the expectation in Algebra I. Students should explore how asymptotes move as this parent function is translated.
- Emphasize reading a graph from left to right when identifying key features such as increasing and decreasing intervals.
- For students to have full understanding of exponential functions, instruction includes MA.912.AR.5.4. Growth or decay of a function can be defined as a key feature (constant percent rate of change) of an exponential function and useful in understanding the relationships between the two.
- Instruction includes the use of x-y notation and function notation.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation
      If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.
- Instruction includes the use of multiple forms of exponential functions, such as $f(x) = b^x$, $f(x) = e^x$ and $f(x) = pe^{kx}$.

Common Misconceptions or Errors

- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.

- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.
- Students may have trouble translating from a written description to mathematical expression.
- Students may identify key features incorrectly if they don't read the graph from left to right.
- Students may get confused with positive infinity when increasing and negative infinity when decreasing.

Instructional Tasks

Instructional Task 1 (MTR.5.1)

Use the two functions below to answer the following questions.

$$f(x) = 1.7^x$$
$$g(x) = 0.6^x$$

Part A. What do you expect $f(0)$ to be? What about $g(0)$?

Part B. As x gets very large, what do you expect the graph of function f to look like? What do you expect the graph of g to look like?

Part C. Graph each function using technology. How do the graphs compare with your answers from Part A and Part B?

Instructional Items

Instructional Item 1

An exponential function is given by the equation $y = \left(\frac{12}{3}\right)^{2x}$. What is the asymptote for the graph?

Instructional Item 2

An exponential function is given.

$$y = 50(1.1)^t$$

Part A. Does this function represent exponential growth or decay?

Part B. What would be the range of the function?

Part C. What would be the y-intercept of the function?

Part D. What is the constant percent rate of change of y with respect to t.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.7

Benchmark

MA.912.AR.5.7 Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context.

Example: The graph of the function $f(t) = e^{5t+2}$ can be transformed into the straight line $y = 5t + 2$ by taking the natural logarithm of the function's outputs.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior and asymptotes.

Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Clarification 3: Instruction includes understanding that when the logarithm of the dependent variable is taken and graphed, the exponential function will be transformed into a linear function.

Clarification 4: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Exponential function
- Function
- Function notation
- Linear function
- Range
- Set builder notation
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.5.3, MA.912.AR.5.5

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students worked with exponential functions in limited forms. In Math for College Algebra, students solve and graph problems modeled with exponential functions, determining key features and constraints in terms of the context.

- Problem types includes creating a function as a tool to determine requested information or providing the graph or function that models the context.
- Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will deepen their understanding of why asymptotes are useful guides to complete the graph of a function. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.
- Growth or decay of a function can be defined as a key feature (constant percent rate of change) of an exponential function and useful in understanding the relationships between two.

- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation
      If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
    - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
    - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
    - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
    - For example, if the range of a function is all real numbers, is can be represented in interval notation as $(-\infty, \infty)$. This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Instruction includes making the connection to constraints (MA.912.AR.9.6) for a given context. Students should develop an understanding that exponential graphs, without context, have no constraints on their domain and range. When specific contexts are modeled by exponential functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.

Common Misconceptions or Errors

- Students may not fully understand how to interpret proper function notation when determining the key features of an exponential function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.
- Students may have a difficult time determining the constraints with fractions.
  - For example, the function $y = m^{-n}$ where $m^{-n} = \frac{1}{m^n}$. In this case, the value of $m \neq 0$, since this would make the function undefined.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

The population of a city is $K = 178{,}563e^{0.011t}$ where $t = 0$ represents the population in the year 2012.

Part A. Find the population of the city in the years 2022 and 2030.
Part B. What would be the domain?
Part C. What is the range?

Instructional Items

Instructional Item 1

An exponential function is given.

$$y = 17^{2m}$$

Part A. Does this function represent exponential growth or decay?
Part B. Select four values for m and create a graph.
Part C. What is the domain for the function?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.8

Benchmark

MA.912.AR.5.8 Given a table, equation or written description of a logarithmic function, graph that function and determine its key features.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.
Clarification 2: Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.3.7
- MA.912.AR.4.4
- MA.912.F.1.1, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Exponential function
- Function
- Function notation
- Logarithmic function
- Range
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

Next Benchmarks

- MA.912.AR.5.9
- MA.912.AR.6.6
- MA.912.AR.9.6

Purpose and Instructional Strategies

In Algebra I, students graph exponential functions and determine their key features, including asymptotes and end behavior. In Math for College Algebra, students develop an understanding of the inverse relationship of exponential and logarithmic functions.

- Instruction includes making the connection between exponents and logarithms to compare the two in terms of functions (*MTR.5.1*).
- Instruction includes the use of x-y notation and function notation.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation

If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.

- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the x- or y-axis when necessary.
- Instruction includes functions with various bases, including base 10 and base e.

Common Misconceptions or Errors

- Students may not fully understand how to interpret proper function notation when determining the key features of a logarithmic function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Given $f(x) = \log_3(x)$.

Part A. Graph the function.
Part B. State the domain of the function.
Part C. State the range of the function.
Part D. Determine the asymptote(s).

Instructional Items

Instructional Item 1

Below is a table with values for $f(x) = \log_7(x)$

x	$f(x)$
2	2.8074
4	1.4037
6	1.086
8	0.93578
10	0.8451

Part A. Graph the function.
Part B. State the domain of the function.
Part C. State the range of the function.
Part D. Determine the asymptote(s).

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5.9

Benchmark

MA.912.AR.5.9	Solve and graph mathematical and real-world problems that are modeled with logarithmic functions. Interpret key features and determine constraints in terms of the context.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.
Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1, MA.912.NSO.1.2, MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.9.10
- MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.6
- MA.912.F.2

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Exponential function
- Function
- Function notation
- Logarithmic function
- Range
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

Next Benchmarks
- MA.912.AR.7.2, MA.912.AR.7.3, MA.912.AR.7.4
- MA.912.AR.8.2, MA.912.AR.8.3

Purpose and Instructional Strategies

In Algebra I, students solved problems modeled with linear, exponential and quadratic functions. In Math for College Algebra, students solve problems modeled with logarithmic functions.

- Instruction features a variety of real-world contexts. Some of these contexts should require students to create a function as a tool to determine requested information or should provide the graph or function that models the context.
- Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will deepen their understanding of why asymptotes are useful guides to complete the graph of a function. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation.

    - Words
      If the domain is all real numbers, it can be written as “all real numbers” or “any value of x, such that x is a real number.”
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as “all values of y such that y is less than or equal to zero.”

- Interval notation
  If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.

- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
  - For example, if the range of a function is all real numbers, is can be represented in interval notation as $(-\infty, \infty)$. This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.
- Instruction includes differentiating when to use base 10 or natural logs based on the context given.

Common Misconceptions or Errors

- Students may not fully understand how to interpret proper function notation when determining the key features of a logarithm function.
- Students may not fully understand how to use interval notation.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.3.1 and MTR.5.1)

A video uploaded to social media initially had 75 views one minute after it was posted. The total number of views to date has been increasing exponentially and can be modeled by the function $y = 75e^{0.2t}$, where t represents time measured in days since the video was posted.

Part A. Convert the exponential to logarithmic form.
Part B. How many days will it take until 5000 people have viewed this video?
Part C. State the domain for the problem.
Part D. Sketch a graph that represents days 5 – 10.

Instructional Items

Instructional Item 1

Wanda invests \$10,000 in a company with earnings of 4% per year.

Part A. Write an equation in logarithmic form that describes the situation.
Part B. How long will it take to accumulate \$20,000?
Part C. How long will it take to accumulate \$25,000?
Part D. Sketch a graph that represents the amount accumulated from the initial investment to reaching the goal of \$25,000.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.7 *Solve and graph radical equations and functions in one and two variables.*

MA.912.AR.7.1

Benchmark

MA.912.AR.7.1 Solve one-variable radical equations. Interpret solutions as viable in terms of context and identify any extraneous solutions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.3
- MA.912.AR.5.2
- MA.912.AR.8.1

Terms from the K-12 Glossary

- Domain
- Intercept
- Inverse functions

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.1
- MA.912.AR.3.1
- MA.912.AR.4.1

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students solved one variable equations including linear, quadratic and absolute value, and determined if the solution(s) was viable. In Math for College Algebra, students solve one variable equations involving radicals.

- Instruction for solving radical equations will follow many of the same characteristics. One will still need to isolate the one variable you have in order to solve. Students will need to understand the inverse relationship between radicals and roots as well as how to convert between radical form and rational exponent form *(MTR.2.1)*.
- Students will also need to understand laws of exponents and how to utilize them to simplify radical expressions and solve radical equations. Students should feel comfortable finding and representing the solutions to a radical equation algebraically, graphically, and numerically (with a table) *(MTR.2.1)*.
- Students should be able to determine if their solution is viable using multiple representations. Students should also be able to explain how many solutions a radical equation will give them as well as what features of that radical equation will determine how many solutions it has as well as if there are any extraneous solutions *(MTR.6.1)*.

Common Misconceptions or Errors

- Students may not understand how to determine if a solution is extraneous.
- Teachers should present the students with a few examples when extraneous solutions arise and ask the students to look for patterns for when those extraneous solutions arise. Teachers should remind students that solving an equation is finding the values of the variable that make the statement true.
- Students may not completely understand the relationships between roots and radicals so they may use the wrong root or exponent to isolate the variable when solving the equation.
- Student may square an individual term instead of squaring the whole side of the equation when trying to solve a radical equation.
    - For example, in the equation $\sqrt{x}+\sqrt{x+2}=2$, students may square the left term on the left side of the equation and not realize to square whole expression on the left side.
- Students may not understand how to use a graph or table to justify what they have found algebraically.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

Part A. Create a one variable radical equation that would have one solution.

Part B. Explain what would you have to change about the equation to have two solutions.

Part C. Explain what would you have to change about the equation to have no solution.

Instructional Task 2 (MTR.2.1, MTR.4.1)

An equation is given.

$$\sqrt{4x-1}=3$$

Part A. What is/are the solution(s)?

Part B. Describe how would you find the solution(s) on a graph.

Part C. Describe how would you find the solution(s) in a table.

Instructional Task 3 (MTR.4.1, MTR.5.1)

Part A. Explain what it means to get an extraneous solution to a radical equation.

Part B. What features will happen in a radical equation to make an extraneous solution arise?

Part C. Give an example of a radical equation with an extraneous solution.

Instructional Task 4 (MTR.3.1)

One estimate for how far one can see on a clear day is given by the formula $v = 1.225\sqrt{a}$, where v represents the visibility in miles and a represents the altitude in feet.

Part A. A person is parasailing and can see 64 kilometers to the horizon, use the given formula to determine how far above the ground the person parasailing.

Part B. If you were given a value for the altitude rather than the visibility, how would that change our approach to finding a solution? What would the value for a mean in the context?

Instructional Items

Instructional Item 1

Find the solution(s) to the following equation $\sqrt[3]{x-3} = -5$. Explain how you found the solution(s).

Instructional Item 2

Find all of the solution(s) to the following equation $\sqrt[4]{x} = \sqrt[5]{x}$.

Instructional Item 3

Which of these equations has an extraneous solution?

a. $\sqrt{2-x} = x$

b. $\sqrt{5-x} = 2$

c. $\sqrt{x} - 12 = 0$

d. $\frac{\sqrt{4x}}{6} = x$

Instructional Item 4

The formula for a cone of a cone written in terms of the radius is $r = \sqrt{\frac{3V}{\pi h}}$.

Part A. A mound of mulch is piled in the shape of a cone with the height equal to three times the radius. Calculate the volume of such a mound of mulch whose radius equals 2.5 yards.

Part B. Is your solution that you found viable? How do you know?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.8 *Solve and graph rational equations and functions in one and two variables.*

MA.912.AR.8.1

Benchmark

MA.912.AR.8.1	Write and solve one-variable rational equations. Interpret solutions as viable in terms of the context and identify any extraneous solutions.

Benchmark Clarifications:
Clarification 1: Within the Algebra II course, numerators and denominators are limited to linear and quadratic expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.4.2
- MA.912.AR.5.2
- MA.912.AR.7.1

Terms from the K-12 Glossary

- Linear expression
- Quadratic expression
- Rational expression
- Rational number

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.1
- MA.912.AR.3.1
- MA.912.AR.4.1

Next Benchmarks

- MA.912.AR.8.3
- MA.912.C.1.3

Purpose and Instructional Strategies

In Algebra I, students solved one variable equations including linear, quadratic and absolute value, and determined if the solution(s) was viable. In Math for College Algebra, students solve one variable equations involving rational expressions.

- Instruction includes making the connection to operations with fractions (*MTR.5.1*).
  - For example, when solving rational equations that involve addition or subtraction, students can find common denominators to assist in solving.
    - Given the equation $\frac{1}{x-2}+\frac{1}{x+3}=\frac{1}{x}$, students can determine a common denominator of $(x-2)(x+3)$ to rewrite the equation as $\frac{x+3}{(x-2)(x+3)}+\frac{x-2}{(x-2)(x+3)}=\frac{1}{x}$. Students should recognize that since the fractions on the left side of the equation have the same denominator, the equation can be rewritten as $\frac{2x+1}{(x-2)(x+3)}=\frac{1}{x}$. From here students can either solve using proportional reasoning (MA.7.AR.3) or can find common denominators between fractions on either side of the equal side and then set the numerators equivalent to one another.
- Instruction includes how to determine what the solutions to the equation are and whether they are viable (*MTR.6.1*). Students should have experience with rational equations that produce different types of solutions: more than one, exactly one, and extraneous.
- Instruction provides opportunities for students to discuss why extraneous solutions may arise with rational equations *(MTR.4.1)*.
- Instruction gives students the opportunity to discuss constraints and what effect those

constraints have on the solution(s) to the equations *(MTR.4.1)*.

Common Misconceptions or Errors

- Students may struggle to determine if a common denominator is needed to solve a rational equation.
- If students determine a common denominator is needed, they may struggle to decide what factors are needed to be multiplied to the numerator and the denominator while keeping the rational equation equivalent.
- Students may forget to test the solutions in the original equation to determine if it is extraneous or not.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)

Explain how you could find the solution(s) to the rational equation given. Share your strategy with a partner.

$$\frac{3}{x} - \frac{x}{x+1} = \frac{2x}{x(x+1)}$$

Instructional Task 2 (MTR.5.1)

Create a rational equation with the following:

a. One solution
b. Two solutions
c. Infinitely many solutions
d. An extraneous solutions

Instructional Task 3 (MTR.7.1)

At a local high school, sophomores are designing and printing school lanyards for the upcoming freshman class. An online printing company charges $40 set up fee and $2.30 for each printed lanyard.

Part A. Create an equation to represent the average cost $C(x)$, in dollars, per lanyard if l lanyards are printed using this company.

Part B. What is the average cost per lanyard to print 55 lanyards? 105 lanyards?

Part C. How many lanyards should be printed to have an average cost of $2.50 or less per lanyard? Explain how you know.

Instructional Items

Instructional Item 1

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of 30 games against his brother.

Part A. How many games would Chase have to win in a row in order to have a 75% winning record?

Part B. How many games would Chase have to win in a row in order to have a 90% winning record?

Part C. Is Chase able to reach a 100% winning record? Explain why or why not.

Part D. Suppose that after reaching a winning record of 90% in Part B, Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?

Instructional Item 2

An equation is given below. Determine the solution(s), including any extraneous solutions.

$$\frac{2x}{x+3}+\frac{4}{x^2+3x}=\frac{3x^2}{x}$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9 *Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.*

MA.912.AR.9.4

Benchmark

MA.912.AR.9.4 Graph the solution set of a system of two-variable linear inequalities.

Benchmark Clarifications:
Clarification 1: Instruction includes cases where one variable has a coefficient of zero.
Clarification 2: Within the Algebra I course, the system is limited to two inequalities.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5

Terms from the K-12 Glossary

- Inequality

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.6, MA.912.AR.2.7, MA.912.AR.2.8

Next Benchmarks

- MA.912.AR.9.8

Purpose and Instructional Strategies

In Algebra I, students solved systems of linear inequalities limited to two inequalities. In Mathematics for College Algebra, students solve problems involving systems of linear inequalities with two or more linear inequalities.

- For students to have full understanding of systems, instruction includes MA.912.AR.9.4 and MA.912.AR.9.6. Equations and inequalities and their constraints are all related and the connections between them should be reinforced throughout the instruction.
- Instruction includes the use of various forms of linear inequalities.
  - Standard Form
    Can be described by the inequality $Ax+By>C$, where A, B, and C are any rational number and any inequality symbol can be used.
  - Slope-intercept form
    Can be described by the inequality $y\geq mx+b$, where m is the slope and b is the y-intercept and any inequality symbol can be used.
  - Point-slope form
    Can be described by the inequality $y-y_1>m(x-x_1)$, where (x_1,y_1) are a

point on the boundary line and m is the slope of the line and any inequality symbol can be used.

- Include examples where one variable has a coefficient of zero.
  - For example, a system could include the inequalities $x > 5$ and $5x + 3y > 2$ or $y > 2$ and $y < -x$.
- Instruction includes the connection to graphing solution sets of one-variable inequalities on a number line, recognizing whether the boundary line should be dotted (exclusive) or solid (inclusive). Additionally, have students use a test point to confirm which side of the line should be shaded *(MTR.6.1)*. Instruction includes the use of graphing utilities to help students visualize the solution to the system of inequalities.
- Students should recognize that the inequality symbol only directs where the line is shaded (above or below) for inequalities when in slope-intercept form. Students shading inequalities in other forms will need to use a test point to determine the correct half-plane to shade.
- The solution to a system of inequalities is the area where all the shading overlaps. If the areas do not overlap, it has no solution.
- Instruction includes graphing multiple inequalities and identifying points that bound the region made by the solution set.
- Instruction allows students to make connections between the algebraic and graphical representations of inequalities in two variables *(MTR.2.1)*.
- Problem types include linear programming.

Common Misconceptions or Errors

- Students may have difficulties making connections between graphic and algebraic representations of systems of inequalities.
- Students may confuse which points are in the solution set of a system that includes inequalities (including points on the lines in a system of inequalities).
- Students may shade the wrong half-plane or graph the incorrect boundary line (solid vs. dashed).

Instructional Tasks

Instructional Task 1(MTR.4.1, MTR.6.1)

Part A. Graph the solution set to the system of inequalities.

$$y > -x + 4$$
$$y < \frac{2}{5}x + 2$$

Part B. What is one point that is a solution to the system above? Explain how you know it is a solution to the system.

Instructional Task 2 (MTR.6.1)

Part A. Graph the solution set to the system of inequalities.

$$x + y \geq 10$$
$$x \leq 8$$
$$y \geq 6$$

Part B. List the coordinates of the boundary created by the system of inequalities in Part A.

Part C. What are two points, aside from the boundary points you found in Part B that are solutions to the system above?

Instructional Items

Instructional Item 1

Graph the solution set to the system of inequalities below.

$$x \geq 3$$
$$\frac{3}{5}x + y < -3$$

Instructional Item 2

Graph the solution set to the system of inequalities below.

$$-y - 3x \leq -1$$
$$2y \geq 8x - 6$$

Instructional Item 3

Graph the solution set to the system of inequalities below and list the points that form the corners of the solution set.

$$x \geq 0$$
$$y \geq 0$$
$$2x - 3y \leq 6$$
$$y \leq -x + 6$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9.6

Benchmark

MA.912.AR.9.6	Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or nonviable options.

Benchmark Clarifications:

Clarification 1: Instruction focuses on analyzing a given function that models a real-world situation and writing constraints that are represented as linear equations or linear inequalities.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5

Terms from the K-12 Glossary

- Inequality
- Linear equation

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.6, MA.912.AR.2.7, MA.912.AR.2.8

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students represented constraints as systems of linear equations or inequalities and interpreted solutions as viable or non-viable options. In Mathematics for College Algebra, students solve problems involving linear programming with two or more linear equations and/or inequalities, subject to the constraints imposed by the real-world context given *(MTR.7.1)*.

- Instruction includes the use of graphing utilities to help students visualize the solution to the system of equations/inequalities *(MTR.7.1)*.
- For students to have full understanding of systems, instruction includes MA.912.AR.9.4

and MA.912.AR.9.6. Equations and inequalities and their constraints are all related and the connections between them should be reinforced throughout the instruction *(MTR.5.1)*.

- Allow for both inequalities and equations as constraints. Include cases where students must determine a valid model of a function.
    - Students often use inequalities to represent constraints throughout Algebra I. Equations can be thought of as constraints as well. Solving a system of equations requires students to find a point that is constrained to lie on specific lines simultaneously.
- Instruction includes the use of various forms of linear equations and inequalities.
    - Standard Form can be described by the equation $Ax + By = C$, where A, B and C are any rational number and any equal or inequality symbol can be used.
    - Slope-intercept form can be described by the inequality $y \geq mx + b$, where m is the slope and b is the y-intercept and any equal or inequality symbol can be used.
    - Point-slope form can be described by the inequality $y - y_1 > m(x - x_1)$, where $(x_{1,} y\ )$ are a point on the line and m is the slope of the line and any equal or inequality symbol can be used.
- Instruction includes graphing multiple inequalities and identifying points that bound the region made by the solution set. Instruction also includes writing objective functions to maximize or minimize a quantity. An objective function is an algebraic expression in multiple variables that describes a quantity that can be minimized or maximized *(MTR.2.1, MTR.6.1, MTR.7.1)*.

Common Misconceptions or Errors

- Students may have difficulty translating word problems into systems of equations and inequalities.
- Students may shade the wrong half-plane for an inequality.
- Students may graph an incorrect boundary line (dashed versus solid) due to incorrect translation of the word problem.
- Students may not identify the restrictions on the domain and range of the graphs in a system of equations based on the context of the situation. Teachers can encourage the students to discuss the meaning of the restriction in the context of the situation *(MTR.4.1)*.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Bottled drinks and hurricane survival kits are to be transported to survivors of an area who experienced a hurricane. Each container of bottled drinks will serve 15 people and each hurricane survival kit will help 10 people.

Part A. If x represents the number of containers of bottled drinks to be transported and y represents the number of hurricane survival kits, write the objective function that models the number of people that can be helped.

Part B. Each truck can carry no more than 70,000 kilograms. The truck is transporting bottled drinks and hurricane survival kits. The bottled drinks weigh 15 kilograms per container and each hurricane survival kit weights 8 kilograms. If x represents the number of bottled drink containers to be transported and y the number of hurricane survival kits to be transported, write an inequality that represents this constraint.

Part C. Each truck can transport a total volume of supplies that does not exceed 7000 cubic meters. If x represent the number of bottled drink containers to be transported and y the number of hurricane survival kits to be transported, write an inequality that represents this constraint.

Part D. Determine how many containers of bottled drinks and how many hurricane survival kits should be transported on each truck to maximize the number of hurricane survivors that can be assisted.

Instructional Items

Instructional Item 1

Use the given objective function and system of linear inequalities representing constraints that are given to determine the maximum value of the objective function and the values of x and y for which the maximum occurs.

$$f(x,y) = 3x + 2y$$
$$x \geq 0$$
$$y \geq 0$$
$$3x + y \leq 9$$
$$x + y \geq 6$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9.10

Benchmark

MA.912.AR.9.10	Solve and graph mathematical and real-world problems that are modeled with piecewise functions. Interpret key features and determine constraints in terms of the context.

Example: A mechanic wants to place an ad in his local newspaper. The cost, in dollars, of an ad x inches long is given by the following piecewise function. Find the cost of an ad that would be 16 inches long.

$$C(x) = \begin{Bmatrix} 12x & x < 5 \\ 60 + 8(x-5) & x \geq 5 \end{Bmatrix}$$

Benchmark Clarifications:

Clarification 1: Key features are limited to domain, range, intercepts, asymptotes and end behavior.

Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9

Terms from the K-12 Glossary

- Piecewise function

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I students solved problems modeled with various functions including linear, quadratic, exponential and absolute value. In Math for College Algebra students extend their knowledge of these to include piecewise functions.

- Instruction focuses on functions within this course.
- Students should be able to identify key features of graphs that are specific to the function types included in the piecewise defined function that they are given.
- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation.
  - Words
    If the domain is all real numbers, it can be written as “all real numbers” or “any value of x, such that x is a real number.”
  - Inequality notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-builder notation
    If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as “all values of y such that y is less than or equal to zero.”
  - Interval notation
    If the domain is all values of x less than or equal to 3, it can be represented as

$(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.

- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
  - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
  - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
  - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.
  - For example, if the range of a function is all real numbers, is can be represented in interval notation as $(-\infty, \infty)$. This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).
- Students should be given the opportunity to discuss how constraints can be written and adjusted based on the context they are given.
- Students should be given situations and asked to create piecewise functions to represent the given situations. They should also be given piecewise functions and asked to graph and find solutions.
- Students should be able to determine constraints specific to the types of functions included in the piecewise defined function and to the real-world context presented for the piecewise function.

Common Misconceptions or Errors

- Students may have a hard time determining which function in the piecewise function to use when finding the function value at a specified x-value.
- Student may not know how to read the piecewise function notation to graph the function correctly.
- Students may not be able to interpret the key features of the specific graphs within the piecewise defined function.
- Students may not identify the restrictions on the domain and range of the graphs in a system of equations based on the context of the situation. Teachers can encourage the students to discuss the meaning of the restriction in the context of the situation *(MTR.4.1)*.

Instructional Tasks

Instructional Task 1(MTR.2.1, MTR.7.1)

Jessica wants to buy candy from the bulk food store. She can buy sour watermelon gummies for \$3.00 a pound. If she purchases more than three pounds of gummies the price drops by \$.25 a pound. Write a piecewise function and a graph of the function to represent Jessica's situation.

Instructional Task 2 (MTR.2.1, MTR.7.1)

Eric wants to plan a pizza party for the winter sports teams. He is considering pizza prices and found a place that sells small pizzas for $12, medium pizzas for $16, and large pizzas for $20.

- He will choose to order small pizzas if only the wrestling team RSVPs (wrestling team has at most 10 wrestlers).
- He will choose medium pizzas if the wrestling team and soccer teams RSVP (soccer teams have at least 30 players).
- He will order large pizzas if wrestling team, soccer teams, and basketball teams all RSVP (basketball teams have at most 18 players).

Write a piecewise function and a graph of the function to represent Eric's situation. Explain how you determined the constraints for each function.

Instructional Task 3 (MTR.5.1)

Create three piecewise defined functions that contain the points $(1,3)$, $(3,6)$, and $(6,12)$.

Instructional Items

Instructional Item 1

Part A. Graph the function $f(x) = \begin{cases} 4, if -4 \leq x < -2 \\ 2, if -2 \leq x < 0 \\ 1, if\ 0 \leq x < 1 \\ -3, if\ 2 \leq x < 4 \end{cases}$.

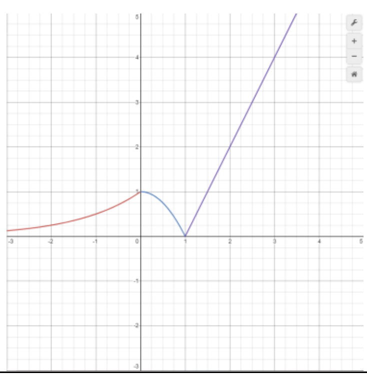
Part A. What is the domain of the function?
Part B. What are the intercepts, if any, of this function?

Instructional Item 2

Graph the function $f(x) = \begin{cases} x^2 + 1, if\ x \geq 0 \\ x + 1, if\ x < -2 \end{cases}$.

Instructional Item 3

Write a piecewise function from the graph given.



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Functions

MA.912.F.1 *Understand, compare and analyze properties of functions.*

MA.912.F.1.1

Benchmark

MA.912.F.1.1	Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

Benchmark Clarifications:
Clarification 1: Within the Algebra I course, functions represented as tables are limited to linear, quadratic and exponential.
Clarification 2: Within the Algebra I course, functions represented as equations or graphs are limited to vertical or horizontal translations or reflections over the x-axis of the following parent functions: $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, $f(x) = \sqrt{x}$, $f(x) = \sqrt[3]{x}$, $f(x) = |x|$, $f(x) = 2^x$ and $f(x) = \left(\frac{1}{2}\right)^x$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.4, MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9

Terms from the K-12 Glossary

- Exponential function
- Function
- Linear function
- Quadratic function
- Reflection
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.F.1

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students identified the domain and range of a relation and determined whether it is a function or not. In Algebra I, students classify function types limited to simple linear, quadratic, cubic, square root, cube root, absolute value and exponential functions. In Mathematics for College Algebra, students will classify function types within real-world context.

- The purpose of this benchmark is to lay the groundwork for students to be able to choose appropriate functions to model real-world data.
- Instruction includes the connection of the graph to its parent function. As various function types are introduced, take time to allow students to produce a rough graph of the parent function from a table of values they develop. Lead student discussion to build connections with why these function types produce their corresponding graphs (*MTR.4.1*).
- Instruction develops the understanding that if given a table of values, unless stated, one cannot absolutely determine the function type, but state which function the table of values could represent. Discuss why function type was selected.
  - For example, if given the function $y = |x|$ and only positive values were given in a table, one could say that table of values could represent a linear or absolute

value function.

Common Misconceptions or Errors

- Some students may miscalculate first and second differences that deal with negative values, especially if they perform them mentally. In these cases, have students quickly write out the subtraction expression (i.e., $-14 - (-2)$) so they can see that they are subtracting a negative value and should convert it to adding a positive value.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Determine the function types of the following

Part A. $f(x) = 3x^2 - 3x - 6$

Part B. $f(x) = m^y$

Part C. $f(x) = 2k + 7$

Part D.

$\boldsymbol{x}$	$\boldsymbol{f(x)}$
0	1600
1	800
2	400
3	200
4	100
5	50

Part E.

$\boldsymbol{x}$	$\boldsymbol{f(x)}$
2	4
3	9
4	16
5	25
6	36
7	49

Instructional Items

Instructional Item 1

Given the table below, determine the function type that could represent it.

x	6	8	10	12	14
y	-1.5	0	2.5	6	10.5

Instructional Item 2

Describe a quadratic function then give an example.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.2

Benchmark

MA.912.F.1.2 Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Algebra I Example: The function $f(x) = \frac{x}{7} - 8$ models Alicia's position in miles relative to a water stand x minutes into a marathon. Evaluate and interpret for a quarter of an hour into the race.

Benchmark Clarifications:
Clarification 1: Problems include simple functions in two-variables, such as $f(x, y) = 3x - 2y$.
Clarification 2: Within the Algebra I course, functions are limited to one-variable such as $f(x) = 3x$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9

Terms from the K-12 Glossary

- Domain
- Function
- Function notation

Vertical Alignment

Previous Benchmarks

- MA.8.F.1

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students work with x-y notation and function notation throughout instruction of linear, quadratic, exponential and absolute value functions. In Math for College Algebra, students continue to use function notation with other function types as well as functions in two variable and perform operations that combine functions, including compositions of functions.

- Instruction leads students to understand that $f(x)$ reads as "f of x" and represents an output of a function in the same way that the variable y represents the output in x-y notation.
- Instruction includes a series of functions with variety of inputs so that students can see the pattern that emerges *(MTR.5.1)*.
  - For example, students can start with $f(x) = 2x^2 + 5x - 7$ and then substitute -2 for x, denoted by $f(-2) = 2(-2)^2 + 5(-2) - 7$. Then, students can substitute a different variable for x, like k, and denote it by $f(k) = 2k^2 + 5k - 7$.
- Students should discover that the number in parenthesis corresponds to the input or x-value on the graph and the number on the other side of the equal sign corresponds to the output or y-value.
- Although not conventional, instruction includes using function notation flexibly.
  - For example, function notation can been seen as $h(x) = 4x + 7$ or $4x + 7 = h(x)$.
- Instruction leads students to consider the practicality that function notation presents to mathematicians. In several contexts, multiple functions can exist that we want to consider simultaneously. If each of these functions is written in x-y notation, it can lead to confusion in discussions.

    - For example, the equations $y = -2x + 4$ and $y = 3x + 7$. Representing these functions in function notation allows mathematicians to distinguish them from each other more easily (i.e., $f(x) = -2x + 4$ and $g(x) = 3x + 7$).
- Function notation also allows for the use of different symbols for the variables, which can add meaning to the function.
    - For example, $h(t) = -16t^2 + 49t + 4$ could be used to represent the height, h, of a ball in feet over time, t, in seconds.
- Function notation allows mathematicians to express the output and input of a function simultaneously.
    - For example, $h(3) = 7$ would represent a ball 7 feet in the air after 3 seconds of elapsed time. This is equivalent to the ordered pair $(3,7)$ but with the added benefit of knowing which function it is associated with.
- When evaluating functions in two variables, students should remember to substitute the values for bath variables before solving.
- A function of two variables has inputs that are ordered pairs (x, y) and the outputs are a single real number. The domain of the function is a set of the ordered pairs and the range is the set of all possible outputs.
    - For example, if you rent a car, the cost depends on two items, the days you keep the car and how far you drive. As a function this would be represented as the cost of the car rental, R, with the inputs of the function being the days driven, d, and the miles driven, m. The function in two variables would be represented as $R(d, m)$ in function notation.

Common Misconceptions or Errors

- Throughout students' prior experience, two variables written next to one another indicate they are being multiplied. That changes in function notation and will likely cause confusion for some of your students. Continue to discuss the meaning of function notation with these students until they become comfortable with the understanding. In other words, (x) does not mean $f \cdot x$.
- Students may need additional support in the order of operations.
- Students may forget to substitute for both variables when evaluating functions in two variables.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

The original value of a painting is $9,000 and the value increases by 7% each year. The value of the painting can be described by the function $V(t) = 9000(1 + 0.07)^t$, where t is the time in years since 1984 and $V(t)$ is the value of the painting.

Part A. Create a table of values that corresponds to this function.

Part B. Graph the function.

Instructional Task 2

Given the function $f\ (x, y) = 3x - 5y + 6$, evaluate for $f(-7,5)$.

Instructional Items

Instructional Item 1

Evaluate $f(24)$, when $f(x) = \frac{3}{2}x + 9$.

Instructional Item 2

Given $f(x) = 2x^4 - 0.24x^2 + 6.17x - 7$, find $f(2)$.

Instructional Item 3

Evaluate $f(a + b, c)$ when $f(x, y) = 6x^2 - 7y + 1$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.3

Benchmark

MA.912.F.1.3 Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.

Benchmark Clarifications:

Clarification 1: Instruction includes making the connection to determining the slope of a particular line segment.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.9.10

Terms from the K-12 Glossary

- Domain
- Exponential function
- Linear function
- Range
- Rate of change
- Slope

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.2
- MA.8.F.1.3

Next Benchmarks

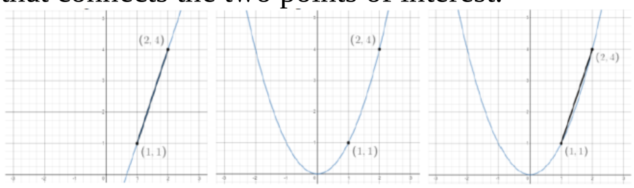
- MA.912.F.1.4
- MA.912.C.2.1, MA.912.C.2.2

Purpose and Instructional Strategies

In Algebra I, students calculated the average rate of change in real-world situations modeled by linear, quadratic, exponential and absolute value functions In Mathematics for College Algebra, students calculate the average rate of change from other functions. In later courses, this concept leads to the difference quotient and differential calculus.

- The purpose of this benchmark is to extend students' understanding of rate of change to allow them to apply it in non-linear contexts.
- Define and discuss the average rate of change and differences between linear and nonlinear functions.
- Instruction emphasizes a graphical context so students can see the meaning of the average rate of change. Students can use graphing technology to help visualize this.

- Starting with the linear function $(x) = 3x - 2$, shown below, ask students to calculate the rate of change between two points using the slope formula. Lead students to verify their calculations visually.
- Once students have successfully used the formula, transition to the graph of $f(x) = x^2$.
- Highlight the same two points and ask students to discuss what the rate of change might be between them *(MTR.4.1)*. Lead students realize that while there is not a constant rate of change, they can calculate an average rate of change for an interval. Show students that this is equivalent to calculating the slope of the line segment that connects the two points of interest.



- Once students have an understanding, ask them to find the average rate of change for other intervals, such as $-2 \leq x \leq -1$ or $0 \leq x \leq 2$. As each of these calculations produce different values, reinforce the concept that nonlinear functions do not have constant rates of change *(MTR.5.1)*.
- Look for opportunities to continue students' work with function notation. Ask students to find the average rate of change between (1) and $f(4)$ for $f(x)$.

Common Misconceptions or Errors

- Some graphs presented to students will only display a certain interval of data. Some students may mistakenly interpret the rate of change for that entire interval rather than a given sub-interval. In these cases, lead students to highlight the domain and range for the sub-interval requested so they can see it more clearly.
- Students may be confused if the average rate of change is 0, even though the function is not constant.
  - For example, the average rate of change of the function $f(x) = x^2$ from $x = -1$ to $x = 1$ is 0.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

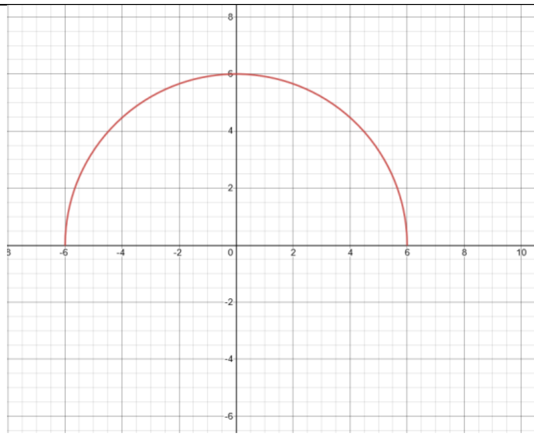
Jorge invests an amount of $5,000 in a money market account at an annual interest rate of 5%, compounded monthly. The function $f(x) = 5000\left(1 + \frac{0.05}{12}\right)^{12x}$, represents the value of the investment after x years.

Part A. Find the average rate of change in the value of Jorge's investment between year 5 and year 7, between year 10 and year 12, and between year 15 and year 17.

Part B. What do you notice about the change in value of the investment over each interval?

Instructional Task 2 (MTR.3.1, MTR.4.1)

The graph of a function is shown.



Part A. Determine the average rate of change on the interval $[-6, -4]$.
Part B. Determine the average rate of change on the interval $[4,6]$.
Part C. Determine the average rate of change on the interval $[-6,6]$.

Instructional Items

Instructional Item 1

Use the table below to find the average gas prices in Florida between 2017 and 2019.

U.S. Energy Information Administration - https://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=pet&s=emm_epmru_pte_sfl_dpg&f=a

Year	Price
2011	$3.479
2012	$3.546
2013	$3.463
2014	$3.302
2015	$2.312
2016	$2.070
2017	$2.330
2018	$2.601
2019	$2.439
2020	$2.070

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1.6

Benchmark

MA.912.F.1.6 Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior and asymptotes.
Clarification 2: Within the Algebra I course, functions other than linear, quadratic or exponential must be represented graphically.
Clarification 3: Within the Algebra I course, instruction includes verifying that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.AR.9.10

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function notation
- Range
- Rate of change
- Slope
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks
- MA.8.AR.3.5
- MA.912.F.1.5, MA.912.F.1.8

Next Benchmarks
- MA.912.F.1.7
- MA.912.DP.2.8
- MA.912.DP.2.9

Purpose and Instructional Strategies

In grade 8, students interpreted the slope and y-intercept of a linear equation in two variables. In Algebra I, students compared key features of two or more linear or nonlinear functions in various representations, with quadratic and exponential function representations limited to graphically only. In Math for College Algebra, students continue their work with comparing key features of nonlinear functions from various representations of functions graphically, algebraically, in tables or written descriptions.

- Within this benchmark, students should recognize that a linear function has a constant rate of change and can be represented by a line and that a nonlinear function does not have a constant rate of change, so it is not a line.
- Instruction includes student exploration of various models to ultimately determine that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
    - Given a table, students will look at the change of y over the change of the x, to check for the slope value. If it’s a linear function it will have a constant slope, but if it’s not it will be a nonlinear function.

$y = x$	
x	y
-1	-1
0	0
1	1
2	2
3	3

Linear = Constant Difference

$y = 2^x$	
x	y
0	1
1	2
2	4
3	8
4	16

Exponential = Constant Factor

$y = x^2$	
x	y
0	0
1	1
2	4
3	9
4	16

Quadratic = Constant Second Difference

  - When looking at a graph, students can easily see visually whether there is a constant rate of change.
  - For example, provide the following context.
    You are being contracted by a large company to provide technical services to a major engineering project. The contract will involve you advising a group of engineers for the three weeks. The company offers you a choice of two methods of payment for your services. The first is to receive \$500 per day of work. The second is to receive payment on a scale: two cents for one total day of work, four cents for two total days, eight cents for three total days, etc. Which method of payment would you choose?
    Have students choose a method of payment and begin a class discussion regarding the reasoning students used to make their choices *(MTR.4.1)*. Ask if students can create a function to represent each payment method *(MTR.7.1)*.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-builder notation
    If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
  - Interval notation
    If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- It is important to have students draw graphs functions on a graphing calculator or computer utility to examine the curves of the graph.
  - For example, as students compare the y-values (output) of an exponential and quadratic function, they notice that an exponential curve will be higher than a quadratic curve.

Common Misconceptions or Errors

- When describing domain or range, students may assign their constraints to the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem.
- Students may also miss the need for compound inequalities when describing domain or range. In these cases, use a graph of the function to point out areas of their constraint that would not make sense in context.
- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable. In these cases, ask reflective questions to help students examine the meaning of the domain and range in the problem. Relate the range of the graph to the dependent variable.

Instructional Tasks

Instructional Task 1(MTR.3.1)

Part A. Sketch a graph for each of the functions shown on the same coordinate plane.

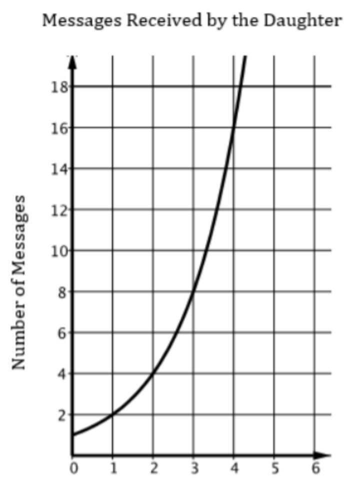
$f(x) = x^2$
$g(x) = |6x + 5|$
$p(x) = 4^{0.45x}$

Part B. For what values of x will exponential function be greater than the absolute and quadratic functions?

Instructional Items

Instructional Item 1

A mother and daughter decided to register for a social media group. The number of messages received by the mother on day n is given by the equation $M = 4n$. The graph shows the number of messages received by the daughter on day n (where n is the number of days since joining the group).



Part A. Explain the difference in the rates of change in the number of messages the mother receives and the number of messages the daughter receives.

Part B. Whose number of daily messages is increasing more rapidly? Provide justification for your answer.

Instructional Item 2

Which equation represents a nonlinear function?

a. $y = 4.3$

b. $y = \pi x$
c. $y = \frac{3}{x}$
d. $y = 2(x - 1)$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.2 *Identify and describe the effects of transformations on functions. Create new functions given transformations.*

MA.912.F.2.1

Benchmark

MA.912.F.2.1 Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of k.

<u>Benchmark Clarifications:</u>
Clarification 1: Within the Algebra I course, functions are limited to linear, quadratic and absolute value.
Clarification 2: Instruction focuses on including positive and negative values for k.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Transformation
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.GR.2
- MA.912.GR.2

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students identify effects of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- In this benchmark, students will examine the impact of transformations on linear, quadratic, exponential, logarithmic and absolute value functions. Instruction includes the use of graphing software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
  - Have students use graphing technology to explore different parent functions.
    - In each graph, toggle on/off the graphs for $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ to examine their impacts on the function. Use the slider to change the value of k (be sure to examine the impacts when k is positive and negative).

        - As students explore, prompt discussion *(MTR.4.1)* among them about the patterns they see as they adjust the slider *(MTR.5.1)*.
- For $f(x) + k$, students should discover that k is being added to the output of the function (equivalent to the y-value) and will therefore result in a *vertical translation* of the function by k units.
    - Ask students to describe what values of k cause the graph to shift up. Which values cause it to shift down?
- For $kf(x)$, students should discover that k is being multiplied by the output of the function (equivalent to the y-value) and will therefore result in a *vertical dilation* (stretch/compression) of the function by a factor of k.
    - Ask students to describe what values of k cause the graph to stretch up vertically. Which values cause it to compress? Which values for k cause the graph to reflect over the x-axis? What is the significance of $k = -1$?
- For $f(x + k)$, students should discover that k is being added to the input of the function and will therefore result in a *horizontal translation* of the function by $-k$ units.
    - Ask students to describe what values of k cause the graph to shift left. Which values cause it to shift right?
- For $f(kx)$, students should discover that k is being multiplied by the input of the function and will therefore result in a *horizontal dilation* (stretch/compression) of the function by a factor of k.
    - Ask students to describe what values of k cause the graph to stretch horizontally. Which values cause it to compress? Which values for k cause the graph to reflect over the y-axis? What is the significance of $k = -1$?
- After students have a good understanding of the impact of $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ on graphs of functions, connect that knowledge to tables of values for a function.
    - For $f(x) + k$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 4$. Guide students to form a table and discuss its connection to the vertical translation observed on the graph.

x	$f(x)$	$f(x) + 4$
1	6	10
2	3	7
3	2	6
4	3	7

    - For $kf(x)$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 0.5$. Guide students to form a table and discuss its connection to the vertical compression observed on the graph.

x	$f(x)$	$0.5[f(x)]$
1	6	3
2	3	1.5
3	2	1
4	3	1.5

    - For $f(x + k)$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 2$. Guide students to form a table and discuss its connection to the horizontal translation observed on the graph. This one may be tricky for students to understand initially. For the table shown, consider $x = 5$. For $f(x)$, $f(5) = 6$, but for $g(x) = f(x+2)$, $g(5) = f(5+2)$, which is equivalent to 18, which is equivalent to shifting $f(7)$ two units to the left on the graph. Bridge this conversation with a graph of the two functions to help them understand the connection.

x	$f(x)$
1	6
2	3
3	2
4	3
5	6
6	11

→

$x+2$	$g(x) = f(x+2)$
3	
4	
5	
6	
7	18
8	27

- For $f(kx)$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 3$. Guide students to form a table and discuss its connection to the horizontal compression observed on the graph.

x	$f(x)$	$3x$	$f(3x)$
1	6	3	
2	3	6	
3		9	
4	3	12	83
5	6	15	146
6		18	258
7	18	21	326
8	27	24	443
9		27	578

Common Misconceptions or Errors

- Similar to writing functions in vertex form, students may confuse effect of the sign of k in $f(x + k)$. Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k. Think about the point on the graph, that makes the argument (value in parentheses), equal zero.
  - For example, for $f(x+4)$, x needs to be -4 to get to zero.
- Vertical stretch/compression can be hard for students to see on linear functions initially and they may interpret stretch/compression as rotation. Introduce the effects of $kf(x)$ and $f(kx)$ by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Students may think that a vertical and horizontal stretch from $kf(x)$ and $f(kx)$ look the same. For linear and quadratic functions, it can help to have a non-zero y-intercept to visualize the difference.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Determine the equation for the graph of $f(x) = x^2$ that has been compressed vertically by a factor of $\frac{1}{3}$.

Instructional Task 2 (MTR.7.1)

A square grass field has a side length of 30 meters. A square pool with the side length of x meters is to be placed in the center.

Part A. Find an equation representing the area of the grass after the pool is installed.

Part B. Graph the relation from the equation in a and adjust your graph for the valid values of x.

Part C. How does the equation and graph change with changes in the area of the field?

Instructional Items

Instructional Item 1

How does the graph of $g(x) = f(x) - 2$ compare to the graph of $f(x) = 3(1.2)^x$?

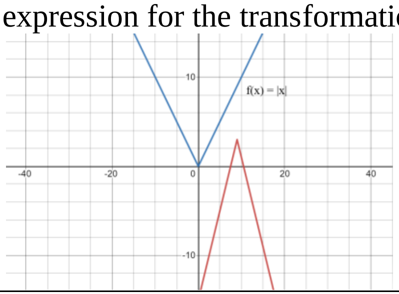
Instructional Item 2

Describe the effect of the transformation $f(2x)$ on the function table below.

x	$f(x)$
-2	4
0	0
2	4
4	16
6	36

Instructional Item 3

Given the absolute value graph of $f(x)$ and its transformation, describe the effects on the parent function and write the expression for the transformation in terms of $f(x)$.



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.2.2

Benchmark

MA.912.F.2.2	Identify the effect on the graph of a given function of two or more transformations defined by adding a real number to the x- or y-values or multiplying the x- or y-values by a real number.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Coordinate plane
- Dilation
- Function notation
- Reflection
- Transformation
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.GR.2
- MA.912.GR.2

Next Benchmarks

- MA.912.AR.7.3
- MA.912.F.1.7

Purpose and Instructional Strategies

In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students identify effects of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- In this benchmark, students will examine the impact of two or more transformations of a given function. Instruction includes the use of graphing software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
- When combining transformation, students will need to consider the order of the transformations.
    - Vertical Transformations affect the output value of the function
        - Given $4f(x) - 2$, vertically shifting by 2 and then vertically stretching by 4 does not create the same graph as vertically stretching by 4 and then vertically shifting by 2, because when we shift first, both the original function and the shift gets stretched, while only the original function gets stretched when we stretch first.
        - When students see an expression such as $4f(x) - 2$, we must first multiply the output value of $f(x)$ by 4, causing a vertical stretch, and then subtract 2, causing the vertical shift. Therefore, students should be reminded that multiplication comes before addition.
    - Horizontal Transformations affects the inputs of the function
        - Given $h(x) = f(5x + 2)$, we must first think about how the inputs to this function h relate to the inputs to function f. For example, if we know $f(5) = 12$, what input to h would produce that output? That means, what value of x will allow $h(x) = f(5x + 2) = 12$. We would need $5x + 2 =$

12, and when solved for x, we would subtract 2, resulting in a horizontal shift, and then divide by 5 which gives us a horizontal compression.

- When students see an expression such as $4f(x) - 2$, we must first multiply the output value of $f(x)$ by 4, causing a vertical stretch, and then subtract 2, causing the vertical shift. Therefore, students should be reminded that multiplication comes before addition.

Combining Vertical Transformations in form $af(x) + k$	First vertically stretch by and then vertically stretch by k
Combining Horizontal Transformations in form $f(bx + h)$	First horizontally shift by h and then horizontally stretch by $\frac{1}{b}$
Combining Horizontal Transformations in form $f(b(x + h))$	First horizontally shift by $\frac{1}{b}$and then horizontally shift by h
Horizontal and Vertical transformation in form $f(x - h) + k$ are independent	The order does not affect the transformation

Common Misconceptions or Errors

- Similar to writing functions in vertex form, students may confuse effect of the sign of k in $f(x + k)$. Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Students may think that a vertical and horizontal stretch from $kf(x)$ and $f(kx)$ look the same. For linear and quadratic functions, it can help to have a non-zero y-intercept to visualize the difference.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

A function is shown.

$$f(x) = 0.25(x - 3)^2 + 5$$

Part A. What transformations occurred to the quadratic parent function $g(x) = x^2$?

Part B. Is there a specific order that the transformations must occur?

Instructional Items

Instructional Item 1

Describe the effect of the transformation $f(x) = 4\sqrt{2 - x}$ on the function table below.

x	$f(x)$
0	0
1	1
2	1.41
3	1.73
4	2

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.2.3

Benchmark

MA.912.F.2.3	Given the graph or table of $f(x)$ and the graph or table of $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$, state the type of transformation and find the value of the real number k.

Benchmark Clarifications:

Clarification 1: Within the Algebra I course, functions are limited to linear, quadratic and absolute value.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Transformation
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.GR.2
- MA.912.GR.2

Next Benchmarks

- MA.912.F.1.7

Purpose and Instructional Strategies

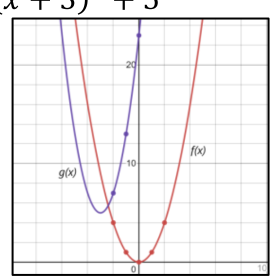
In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students determine the type of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- Instruction includes identifying function transformations involving a combination of translations, dilations and reflections, and determining the value of the real number that defines each of the transformations.
- Transformations can be either horizontal (changes to the input: x) or vertical (changes to the output: $f(x)$).
- There are three different types of transformations: translations, dilations, and reflections.
  - By combining single transformations, a parent function can become a more advanced function.

$$f(x) = x \rightarrow h(x) = Af(B(x + C) + D)$$

Example: $f(x) = x^2 \rightarrow g(x) = 2(x + 3)^2 + 5$

x	$f(x) = x^2$	$g(x) = 2(x + 3)^2 + 5$
-2	4	7
-1	1	13
0	0	23
1	1	37
2	4	55



- Using a graphing utility can help students understand how changing the value of the real numbers in the function's equation change its graph.
- Encourage student's discussion about the effects of changing the value of the real numbers A, B, C, and D in the equation of the function. Ask them to generalize their findings.

Translations	$x \rightarrow x + C$	Shift to the left by C units
	$x \rightarrow x - C$	Shift to the right by C units
	$f(x) \rightarrow f(x) + D$	Shift up by D units
	$f(x) \rightarrow f(x) - D$	Shift down by D units
Dilations	$x \rightarrow Bx;\ B > 1$	Dilates horizontally with scale factor $\frac{1}{B}$ (horizontal stretching)
	$x \rightarrow \frac{1}{B}x;\ B > 1$	Dilates horizontally with scale factor B (horizontal shrinking)
	$f(x) \rightarrow Af(x);\ A > 1$	Dilates vertically with scale factor A (vertical stretching)
	$f(x) \rightarrow \frac{1}{A}f(x);\ A > 1$	Dilates vertically with scale factor $\frac{1}{A}$ (vertical shrinking)
Reflections	$x \rightarrow -x$	Reflects over the y-axis
	$f(x) \rightarrow -f(x)$	Reflects over the x-axis

Common Misconceptions or Errors

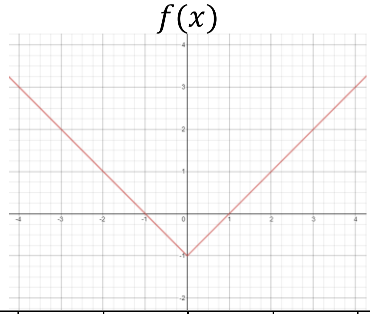
- Some students may have difficulty seeing the impact of a transformation when comparing tables and graphs. In these cases, encourage students to convert the graph to a second table, using the same domain as the first table. This should aid in comparisons.
- Similar to writing functions in vertex form, students may confuse effect of the sign of k in $f(x + k)$. Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Vertical stretch/compression can be hard for students to see on linear functions initially and they may interpret stretch/compression as rotation. Introduce the effects of $kf(x)$ $and\ f(kx)$ by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Students may think that a vertical and horizontal stretch from $kf(x)$ and $f(kx)$ look the same. For linear and quadratic functions, it can help to have a non-zero y-intercept to visualize the difference.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

A graph and table, which represents an absolute value function, are shown below. Describe and determine the value of the real number that defines the transformation from $f(x)$ to $g(x)$.

$f(x)$



x	-2	-1	0	1	2
$g(x)$	2	0	-2	0	2

Instructional Task 2 (MTR.3.1)

Describe the transformations that maps the function $f(x) = 2^x$ to each of the following.

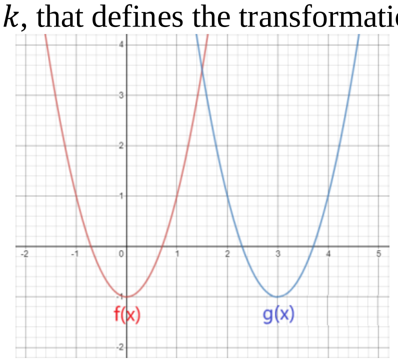
$f(x) = 2^x - 2$ $\quad\quad$ $g(x) = 4^x + 3$

$h(x) = 2^{x-3}$ $\quad\quad$ $b(x) = 3(2^{x+1}) + 2$

Instructional Items

Instructional Item 1

Considering the graph of $f(x)$ and $g(x)$ below, describe the transformation and determine the value of the real number, k, that defines the transformation from $f(x)$ to $g(x)$.



Instructional Item 2

Considering the table below, describe the transformation and determine the value of the real number, k, that defines the transformation from $f(x)$ to $g(x)$.

x	$f(x)$	$g(x)$
1	-9	-8.4
2	-13.2	-12.6
3	-17.4	-16.8
4	-21.6	-21

MA.912.F.2.4

Benchmark

MA.912.F. 2.4	Given the graph or table of values of two or more transformations of a function, state the type of transformation and find the values of the real number that defines the transformation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Transformation
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.GR.2
- MA.912.GR.2

Next Benchmarks

- MA.912.F.1.7

Purpose and Instructional Strategies

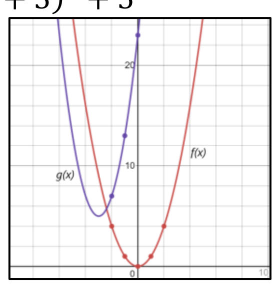
In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students determine the type of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- Instruction includes identifying function transformations involving a combination of translations, dilations and reflections, and determining the value of the real number that defines each of the transformations.
- Transformations can be either horizontal (changes to the input: x) or vertical (changes to the output: $f(x)$).
- There are three different types of transformations: translations, dilations and reflections.
- By combining single transformations, a parent function can become a more advanced function.

$$f(x) = x \rightarrow h(x) = Af(B(x + C) + D)$$

Example: $f(x) = x^2 \rightarrow g(x) = 2(x + 3)^2 + 5$

x	$f(x) = x^2$	$g(x) = 2(x + 3)^2 + 5$
-2	4	7
-1	1	13
0	0	23
1	1	37
2	4	55



- Using a graphing utility can help students understand how changing the value of the real numbers in the function's equation change its graph *(MTR.5.1)*.

- Encourage student's discussion about the effects of changing the value of the real numbers A, B, C, and D in the equation of the function *(MTR.4.1)*. Ask them to generalize their findings.

Translations (addition/subtraction)	$x \to x + C$	Shift to the left by C units
	$x \to x - C$	Shift to the right by C units
	$f(x) \to f(x) + D$	Shift up by D units
	$f(x) \to f(x) - D$	Shift down by D units
Dilations (multiplication/division)	$x \to Bx$; $B > 1$	Dilates horizontally with scale factor $\frac{1}{B}$ (horizontal stretching)
	$x \to \frac{1}{B}x$; $B > 1$	Dilates horizontally with scale factor B (horizontal shrinking)
	$f(x) \to Af(x)$; $A > 1$	Dilates vertically with scale factor A (vertical stretching)
	$f(x) \to \frac{1}{A}f(x)$; $A > 1$	Dilates vertically with scale factor $\frac{1}{A}$ (vertical shrinking)
Reflections (negation)	$x \to -x$	Reflects over the y-axis
	$f(x) \to -f(x)$	Reflects over the x-axis

- Instruction includes determining the order for performing two or more transformations to a single function. A different order of transformations may result in a different outcome.
  - It is helpful to relate the order of the transformations to the order of operations.
    - Horizontal shifts
    - Vertical stretching or shrinking
    - Reflecting
    - Vertical shift

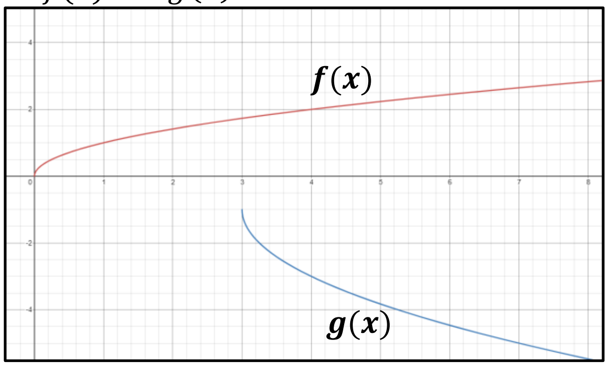
Common Misconceptions or Errors

- Some students may have difficulty seeing the impact of a transformation when comparing tables and graphs. In these cases, encourage students to convert the graph to a second table, using the same domain as the first table. This should aid in comparisons *(MTR.2.1)*.
- Similar to writing functions in vertex form, students may confuse the effect of the sign of k in $(x + k)$. Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of k.
- Vertical stretch/compression can be hard for students to see on linear functions initially and they may interpret stretch/compression as rotation. Introduce the effects of $k(x)$ and $f(kx)$ by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Students may think that a vertical and horizontal stretch from $k(x)$ and $f(kx)$ look the same. For linear and quadratic functions, it can help to have a non-zero y-intercept to visualize the difference.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Below are the graphs of $f(x)$ and $g(x)$.



Part A. Describe the sequence of transformations that map the graph of $f(x)$ to the graph of $g(x)$.

Part B. If $f(x) = \sqrt{x}$, write an equation for $g(x)$.

Instructional Task 2 (MTR.3.1, MTR.5.1)

The tables below show a sequence of transformation for function f.

x	$f(x)$
6	10
12	14
18	15
24	17

Step 1

x	$f(Bx)$
2	10
4	14
6	15
8	17

Step 2

x	$Af(Bx)$
2	20
4	28
6	30
8	34

Step 3

x	$g(x) = Af(Bx) + D$
2	21
4	29
6	31
8	35

Part A. Describe the transformation of $f(x)$ represented in Step 1. State the value of B.

Part B. Describe the transformation of $f(x)$ represented in Step 2. State the value of A.

Part C. Describe the transformation of $f(x)$ represented in Step 3. State the value of D.

Part D. Write an equation for $g(x)$ in terms of $f(x)$.

Instructional Items

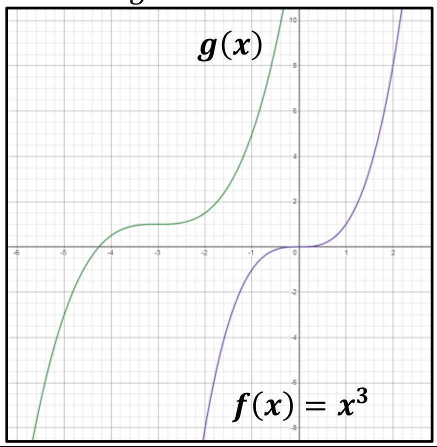
Instructional Item 1

Given the table below, determine the value of A and D.

x	$f(x) = \|x\|$	$g(x) = A\|x\| + D$
-2	2	-4
-1	1	-1
0	0	2
1	1	-1
2	2	-4

Instructional Item 2

In the graph below, function g is obtained from function f through a sequence of transformations. Find an equation for g.



**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.2.5

Benchmark

MA.912.F.2.5	Given a table, equation or graph that represents a function, create a corresponding table, equation or graph of the transformed function defined by adding a real number to the x- or y- values or multiplying the x- or y- values by a real number.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Transformation
- Translation

Vertical Alignment

Previous Benchmarks

- MA.8.GR.2
- MA.912.GR.2

Next Benchmarks

- MA.912.F.1.7

Purpose and Instructional Strategies

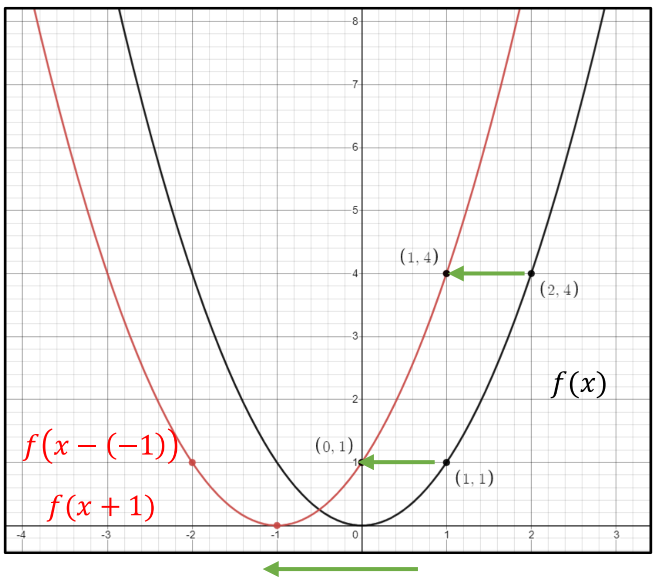
In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identified the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students performed multiple transformations on two-dimensional figures. In Mathematics for College Algebra, students identify effects of transformations on linear, quadratic, exponential, logarithmic and absolute value functions.

- In this benchmark, students will create a table, equation or graph of a transformed function defined by adding a real number to the x- or y- values or multiplying the x- or

y- values by a real number.

- Instruction includes the use of a graphic software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
- Given a function f, the transformed function $g(x) = f(x - C)$ is a horizontal shift of $f(x)$. Adding a real number, C, to all the inputs (x-values) of a function will result in shifting the output left or right depending on the sign of C. If C is positive, the graph will shift right. If C is negative, the graph will shift left.

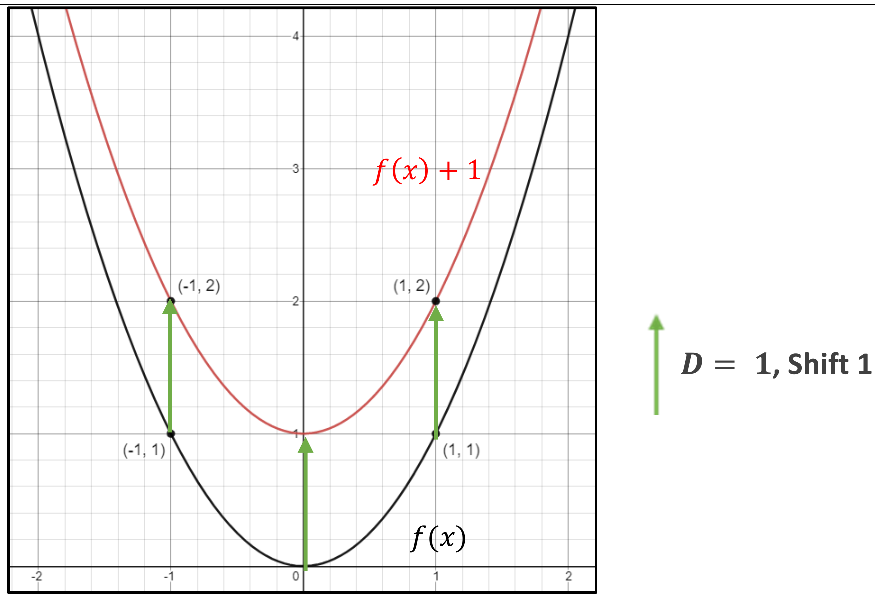
$\boldsymbol{x}$	$(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$	$\boldsymbol{f}(\boldsymbol{x})$	$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x} - \boldsymbol{C}); \boldsymbol{C} = -\boldsymbol{1}$	$(\boldsymbol{x}, \boldsymbol{g}(\boldsymbol{x}))$	$\boldsymbol{g}(\boldsymbol{x})$
-2	$(-2, 4)$	4	$g(-2) = f(-2+1) = f(-1) = 1$	$(-2, 1)$	1
-1	$(-1, 1)$	1	$g(-1) = f(-1+1) = f(0) = 0$	$(-1, 0)$	0
0	$(0, 0)$	0	$g(0) = f(0+1) = f(1) = 1$	$(0, 1)$	1
1	$(1, 1)$	1	$g(1) = f(1+1) = f(2) = 4$	$(1, 4)$	4
2	$(2, 4)$	4	$g(2) = f(2+1) = f(3) =?$	$(2, ?)$	



$\boldsymbol{C = -1}$**, Shift 1 unit to the left**

- Given a function f, the transformed function $g(x) = f(x) + D$ is a vertical shift of $f(x)$. Adding a real number, D, to all the outputs (y-values) of a function will result in shifting the output up or down depending on the sign of D. If D is positive the graph will shift up, and if D is negative the graph will shift down.

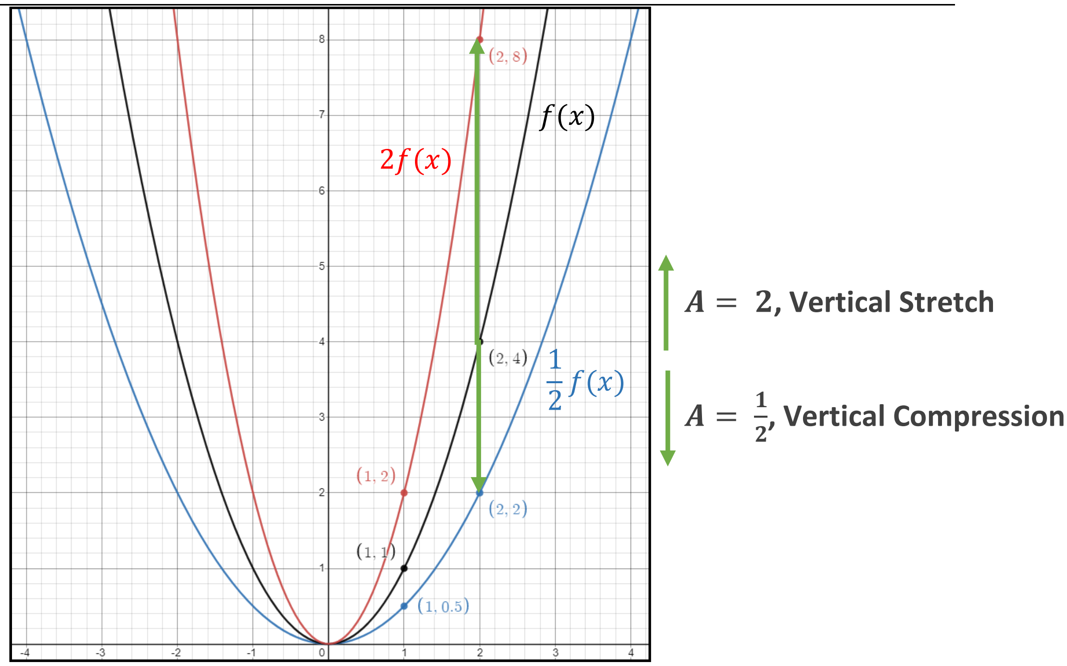
$\boldsymbol{x}$	$(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$	$\boldsymbol{f}(\boldsymbol{x})$	$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{D}; \boldsymbol{D} = \boldsymbol{1}$	$(\boldsymbol{x}, \boldsymbol{g}(\boldsymbol{x}))$	$\boldsymbol{g}(\boldsymbol{x})$
-2	$(-2, 4)$	4	$g(-2) = f(-2) + 1 = 5$	$(-2, 5)$	5
-1	$(-1, 1)$	1	$g(-1) = f(-1) + 1 = 2$	$(-1, 2)$	2
0	$(0, 0)$	0	$g(0) = f(0) + 1 = 1$	$(0, 1)$	1
1	$(1, 1)$	1	$g(1) = f(1) + 1 = 2$	$(1, 2)$	2
2	$(2, 4)$	4	$g(2) = f(2) + 1 = 5$	$(2, 5)$	5



- Discuss with the students that as well as translations of two-dimensional figures, adding a constant to either the input or output of a function change the position of the graph, but it doesn't change the shape of the graph *(MTR.4.1)*.
- Given a function f, the transformed function $g(x) = Af(x)$ is a vertical stretch or compression of $f(x)$. Multiplying all the outputs (y-values) of a function by a real number, A, will result in a vertical stretching or compression depending on the value of A. If A is between 0 and 1 $(0 < A < 1)$, the graph will be vertically compressed and if A is greater than 1 $(A > 1)$, the graph will be vertically stretched.
- If A is a negative number $(A < 0)$, the transformed graph will be a combination of a vertical stretch or compression and a reflection over the x-axis. Discuss with students how multiplying all the y-values by -1 is the same as reflecting a two-dimensional figure over the x-axis *(MTR.4.1)*.

$\boldsymbol{x}$	$\boldsymbol{(x, f(x))}$	$\boldsymbol{f(x)}$	$\boldsymbol{g(x) = Af(x); A = 2}$	$\boldsymbol{(x, g(x))}$	$\boldsymbol{g(x)}$
-2	$(-2, 4)$	4	$g(-2) = 2f(-2) = 8$	$(-2, 8)$	8
-1	$(-1, 1)$	1	$g(-1) = 2f(-1) = 2$	$(-1, 2)$	2
0	$(0, 0)$	0	$g(0) = 2f(0) = 0$	$(0, 0)$	0
1	$(1, 1)$	1	$g(1) = 2f(1) = 2$	$(1, 2)$	2
2	$(2, 4)$	4	$g(2) = 2f(2) = 8$	$(2, 8)$	8

$\boldsymbol{x}$	$\boldsymbol{(x, f(x))}$	$\boldsymbol{f(x)}$	$\boldsymbol{g(x) = Af(x); A = \frac{1}{2}}$	$\boldsymbol{(x, g(x))}$	$\boldsymbol{g(x)}$
-2	$(-2, 4)$	4	$g(-2) = 2f(-2) = 2$	$(-2, 2)$	2
-1	$(-1, 1)$	1	$g(-1) = 2f(-1) = 0.5$	$(-1, 0.5)$	0.5
0	$(0, 0)$	0	$g(0) = 2f(0) = 0$	$(0, 0)$	0
1	$(1, 1)$	1	$g(1) = 2f(1) = 0.5$	$(1, 0.5)$	0.5
2	$(2, 4)$	4	$g(2) = 2f(2) = 2$	$(2, 2)$	2



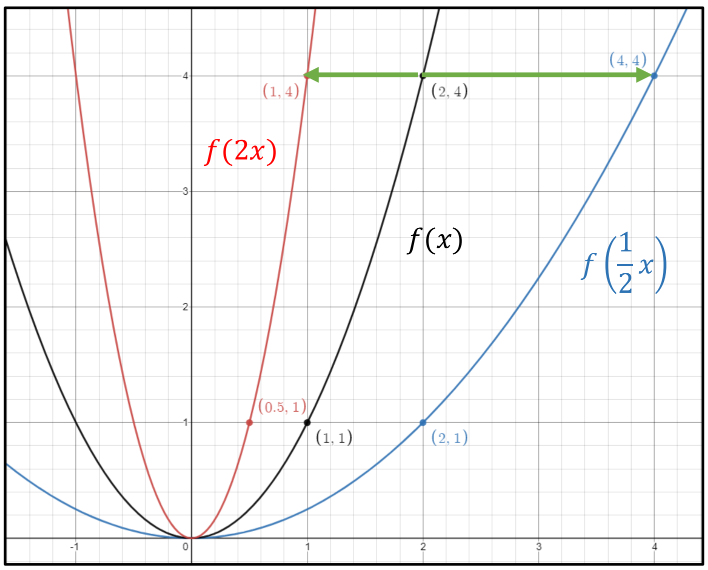
- Given a function f, the transformed function $g(x) = f(Bx)$ is a horizontal stretch or compression of $f(x)$. Multiplying all the inputs (x-values) of a function by a real number, B, will result in a horizontal stretching or compression depending on the value of B. If B is between 0 and 1 $(0 < B < 1)$, the graph will be horizontally stretched by $\frac{1}{B}$ and if B is greater than 1 $(B > 1)$, the graph will be horizontally compressed by $\frac{1}{B}$.
- If B is a negative number $(B < 0)$, the transformed graph will be a combination of a horizontal stretch or compression and a reflection over the y-axis. Discuss with students how multiplying all the x-values by -1 is the same as reflecting a two-dimensional figure over the y-axis *(MTR.4.1)*.

$\boldsymbol{x}$	$(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$	$\boldsymbol{f}(\boldsymbol{x})$		$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{Bx}); \boldsymbol{B} = \mathbf{2}$	$(\boldsymbol{x}, \boldsymbol{g}(\boldsymbol{x}))$	$\boldsymbol{g}(\boldsymbol{x})$
-4	$(-4, 16)$	16		$g(-4) = f(-8) = ?$		
-2	$(-2, 4)$	4		$g(-2) = f(-4) = 16$	$(-2, 16)$	16
-1	$(-1, 1)$	1		$g(-1) = f(-2) = 4$	$(-1, 4)$	4
0	$(0, 0)$	0		$g(0) = f(0) = 0$	$(0, 0)$	0
1	$(1, 1)$	1		$g(1) = f(2) = 4$	$(1, 4)$	4
2	$(2, 4)$	4		$g(2) = f(4) = 16$	$(2, 16)$	16
4	$(4, 16)$	16		$g(4) = f(8) = ?$		

- Discuss with students the meaning of $g(x) = f(2x)$. In this case, the output value, $g(x)$, is the same as the output value of $f(x)$ at an input that is twice the size.

x	$(x, f(x))$	$f(x)$	$g(x) = f(Bx); B = \frac{1}{2}$	$(x, g(x))$	$g(x)$
-4	$(-4, 16)$		$g(-4) = f(-2) = 4$	$(-4, 4)$	
-2	$(-2, 4)$		$g(-2) = f(-1) = 1$	$(-2, 1)$	
-1	$(-1, 1)$		$g(-1) = f\left(-\frac{1}{2}\right) =?$		
0	$(0, 0)$		$g(0) = f(0) = 0$	$(0, 0)$	
1	$(1, 1)$		$g(1) = f\left(\frac{1}{2}\right) =?$		
2	$(2, 4)$		$g(2) = f(1) = 1$	$(2, 1)$	
4	$(4, 16)$		$g(4) = f(2) = 4$	$(4, 4)$	

- Discuss with students the meaning of $g(x) = f\left(\frac{1}{2}x\right)$. In this case the output value, $g(x)$ is the same as the output value of $f(x)$ at an input that is half the size. Example: $g(4) = f\left(\frac{1}{2} \cdot 4\right) = f(2) = 4$ *(MTR.4.1)*.



$B = 2$, **Horizontal Compression**

$B = \frac{1}{2}$, **Horizontal Stretch**

Common Misconceptions or Errors

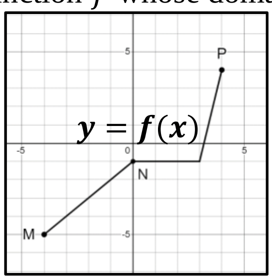
- Some students may have difficulty seeing the impact of a transformation when comparing tables and graphs. In these cases, encourage students to convert the graph to a second table, using the same domain as the first table. This should aid in comparisons *(MTR.2.1)*.
- Some students misinterpret how the parameters of the equation of a transformed function are affected by a horizontal translation. This may indicate that students do not understand the relationship between the graph and the equation of the function.

- For example, a student may think that $g(x) = f(x+1)$ is a horizontal translation to the right because of the positive addend for x. One potential teaching strategy would be using a graphing utility to graph the function $f(x) = (x - C)^2$ creating C as slider, and then allowing students to explore the translation results as the value of the slider changes.
- Some students may have difficulties understanding that multiplying the input of a function by a number greater than 1 will result in a horizontal compression of the graph instead of a stretching. It is important to point out that multiplying the x-value does not change the original value of the input. Because the input is being multiplied by a number greater than 1, a smaller input in the transformed function is needed to obtain the same output from the original function. One potential teaching strategy would be using a graphing utility to graph the function $f(x) = (Bx)^2$ creating B as slider, and then allowing students to explore the stretching/compression results as the value of the slider changes from 0 to 2. Remind students that negative values of B will result in a vertical reflection of the function.

Instructional Tasks

Instructional Task 1 (MTR.2.1)

The figure shows the graph of a function f whose domain is the interval $-4 \le x \le 4$.



Part A: Sketch the graph of each transformation described below and compare it with the graph of f. Explain what you see.

a. $g(x) = f(x) + 2$
b. $h(x) = f(x + 2)$
c. $k(x) = 2f(x)$
d. $r(x) = f(2x)$

Part B: The points labeled M, N, P on the graph of f have coordinates $M = (-4, -5)$, $N = (0, -1,)$ and $P = (-4, 4)$. Complete the table below with the coordinates of the points corresponding to M, N, P on the graphs of g, h, k and r?

$\boldsymbol{f(x)}$	$\boldsymbol{g(x)}$	$\boldsymbol{h(x)}$	$\boldsymbol{k(x)}$	$\boldsymbol{r(x)}$
$(-4, -5)$				
$(0, -1,)$				
$(-4, 4)$				

Instructional Items

Instructional Item 1 (MTR.3.1)

Given the function $f(x) = |x|$, graph the function $f(x)$ and the transformation $g(x) = f(x - 3)$ on the same axes. What do you notice about the x-intercepts of $g(x)$?

Instructional Item 2 (MTR.3.1)

Given the function $f(x) = \log x$, graph the function $f(x)$ and the transformation $g(x) = 3f(x)$ on the same axes. Describe the transformed function, $g(x)$, as it relates to the graph of $f(x)$.

Instructional Item 3 (MTR.3.1)

A function $f(x)$ is given. Create a table for the functions below

a. $g(x) = f(x) + 5$
b. $h(x) = f(2x)$

$\boldsymbol{x}$	2	4	6	8
$\boldsymbol{f(x)}$	1	3	7	11

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3 *Create new functions from existing functions.*

MA.912.F.3.2

Benchmark

MA.912.F.3.2	Given a mathematical or real-world context, combine two or more functions, limited to linear, quadratic, exponential and polynomial, using arithmetic operations. When appropriate, include domain restrictions for the new function.

Benchmark Clarifications:
Clarification 1: Instruction includes representing domain restrictions with inequality notation, interval notation or set-builder notation.
Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.4
- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.1

Terms from the K-12 Glossary

- Domain
- Function
- Function notation

Vertical Alignment

Previous Benchmarks

- MA.7.AR.1
- MA.8.AR.1
- MA.912.AR.1.4, MA.912.AR.1.7

Next Benchmarks

- MA.912.C.2.3, MA.912.C.2.4

Purpose and Instructional Strategies

In middle grades, students performed operations on linear expressions. In Algebra I, students performed addition, subtraction, multiplication and division with polynomials. In Math for College Algebra, students continue to use function notation and they combine functions using arithmetic operations.

- When appropriate, the domain restrictions will be determined for the new function. Students will evaluate the solution when combining two functions for a provided input.
- In this benchmark, students will combine functions through addition, subtraction, multiplication, and division. This process will utilize their prior experience with polynomial arithmetic in MA.912.AR.1 from Algebra I.
    - In mathematical contexts, combinations through addition may be represented as $(f+g)(x)$ or as $h(x)=f(x)+g(x)$.
    - In mathematical contexts, combinations through subtraction should be represented as $(f-g)(x)$ or as $h(x)=f(x)-g(x)$.
    - In mathematical contexts, combinations through multiplication should be represented as $(f \cdot g)(x)$ or as $h(x)=f(x) \cdot g(x)$.
    - In mathematical contexts, combinations through division should be represented as $\left(\frac{f}{g}\right)(x)$ or as $h(x)=\frac{f(x)}{g(x)}$ where $g(x) \neq 0$. Additionally for division, it can be represented using the division symbol.
- Explain that when functions are combined using addition, subtraction, and multiplication, the domain of the resulting function is only the inputs (x-values) that are common to the domains of the original functions. The domain of $f+g$, $f-g$ and $f \cdot g$ is the intersection of the domains of f and g.
- When combining functions by division, students will need to consider values in the domain of the quotient that should be restricted. Values that should be restricted are those that would cause the denominator to equal zero.
- Instruction in Algebra I included representing domain, range and constraints using words, inequality notation and set-builder notation. In Math for College Algebra, instruction also includes interval notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x>2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation
      If the domain is all values of x less than or equal to 3, it can be represented as

$(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.

- Instruction may include the use of graphs and tables to foster a deeper understanding of operations with functions.
- Students have previous knowledge of linear and quadratic functions, and should make the connection that by multiplying two linear functions, it results in a quadratic function.
  - For example, in order to determine revenue, one could multiply the function that represents price of a product by the function that represents quantity of a product that will be sold at that price.

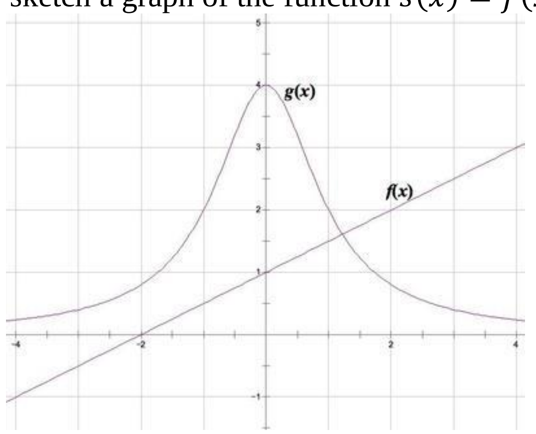
Common Misconceptions or Errors

- When multiplying and dividing functions students may struggle working with exponents, remind students of the multiplication and division properties of exponents, include negative exponents.
- When finding the domain of the new function, students find the union of the domains instead of the intersection. Emphasize that they need to find what the domains have in common.
- When finding the domain of $\frac{f}{g}$ students forget to identify the restrictions on the domain. Remind students that they always need to find the values that make $g(x) = 0$ and exclude them from the domain of $\frac{f}{g}$.

Instructional Tasks

Instructional Task 1 (MTR.5.1)

Using the graphs below, sketch a graph of the function $s(x) = f(x) + g(x)$.



Instructional Items

Instructional Item 1

Given $f(x) = x^2 + 2x + 4$ and $g(x) = -2x + 5$, find $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$, and $\frac{f(x)}{g(x)}$. Determine the domain for each function.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3.4

Benchmark

MA.912.F.3.4 Represent the composition of two functions algebraically or in a table. Determine the domain and range of the composite function.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.8, MA.912.AR.5.9

Terms from the K-12 Glossary

- Domain
- Function
- Function notation

Vertical Alignment

Previous Benchmarks

Next Benchmarks

- MA.912.C.2.3, MA.912.C.2.4

Purpose and Instructional Strategies

In Algebra I, students work with x-y notation and function notation throughout instruction of linear, quadratic, exponential and absolute value functions. In Math for College Algebra, students continue to use function notation with other function types and combine functions, including compositions of functions.

- Define function composition and demonstrate composing functions with several examples. Explain that function composition is an operation that can be performed on two functions so the output of one function becomes the input of another to form a new function. The resulting function is known as a composite function.
- Introduce the notation for composition of functions as $(f \circ g)(x) = f(g(x))$ and explain that we read the left-hand side as "f composed with g at x," and the right-hand side as "f of g of x."
- Emphasize the difference between $f(g(x))$ and $g(f(x))$ and explain that function composition is not commutative. Explain that it is important to follow the order of operations when evaluating a composite function, we evaluate the inner function first.
- Instruction includes determining the domain and range of the composite function. Explain that the domain of the composite function is all inputs x, such that x is in the domain of g and $g(x)$ is in the domain of f.

Common Misconceptions or Errors

- Students sometimes believe that variables represent just a fixed number, so they struggle understanding that a variable can also represent a function.
- Students struggle evaluating functions for numerical inputs, so it is more difficult for them evaluating for inputs that are functions.
- Students may not understand the function notation for composition thinking that $f\big(g(x)\big)$ means to multiply function g by function f.
- Students confuse the composition $(f \circ g)$ with the product $(f \cdot g)$. Emphasize that the composition means "evaluate g at x, then evaluate f at the result $g(x)$."
- Some students might thing that $(f \circ g)$ is the same as $(g \circ f)$. Emphasize that composition is not commutative.
- When finding the domain of a composite function, sometimes students find the union of the domains instead of the intersection. Emphasize that they need to find what the

domains have in common, the intersection. Also, students sometimes forget to include the restrictions on the domain of the inner function on the domain of the composite function. Remind them to use the most restrictive domain.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

Your parents purchased a new reclining chair for the living room. The chair will be delivered to your home. The cost of the chair is d dollars, the tax rate is 6.5%, and the delivery fee is \$50.

Part A. Write a function $f(d)$ for the purchase amount, d, and the delivery fee. State the domain.

Part B. Write another function $g(d)$ for the cost of the chair after taxes. State the domain.

Part C. Write the function $f(g(d))$ and interpret its meaning. State the domain of $f(g(d))$.

Part D. Write the function $g(f(d))$ and interpret its meaning. State the domain of $g(f(d))$.

Part E. Which results in a lower cost to you, $f(g(d))$ or $g(f(d))$? Explain why.

Instructional Task 2 (MTR.7.1)

Let f be the function that assigns to a temperature in degrees Celsius its equivalent in degrees Fahrenheit. Let g be the function that assigns to a temperature in degrees Kelvin its equivalent in degrees Celsius.

Part A. Explain what x and $f\left(g(x)\right)$ represent in terms of temperatures, or explain why there is no reasonable representation.

Part B. Explain what x and $g\left(f(x)\right)$ represent in terms of temperatures, or explain why there is no reasonable representation.

Part C. Given that $f(x) = \frac{9}{5}x + 32$ and $g(x) = x - 273$, find an expression for $f\left(g(x)\right)$.

Part D. Find an expression for the function h which assigns to a temperature in degrees Fahrenheit its equivalent in degrees Kelvin.

Instructional Items

Instructional Item 1 (MTR.3.1)

Given $f(x) = 3x^2 + x + 4$ and $g(x) = x + 4$.

a) Determine $f \circ g$.
b) Determine $g \circ f$.
c) Determine the domain of each composite function.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3.6

Benchmark

MA.912.F.3.6 Determine whether an inverse function exists by analyzing tables, graphs and equations.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.6

Terms from the K-12 Glossary

- Composition of functions
- Inverse of a function

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students analyzed linear, quadratic, exponential and absolute value functions. In Mathematics for College Algebra, extend their knowledge of analyzing functions to determine whether an inverse function exists by analyzing tables, graphs, and equations *(MTR.2.1)*.

- Students will need to use what they have learned about key features and characteristics and patterns that are evident with each function type to determine whether the functions they see represented in graphs, tables, and/or equations are functions that are invertible *(MTR.5.1)*.
- When dealing with a graph, students can use the horizontal line test to determine if an inverse to the function exists. The horizontal line test allows a student to check if a function is one-to-one, meaning there is only one x-value for each y-value. In order to use the horizontal line test, students can draw a horizontal line and check to see if it intersects the graph of the function in all places at exactly one point. If it does, then the given function should have an inverse that is also a function.
- Instruction includes discussing what adjustments could be made to the function to make it where an inverse to the function will exist *(MTR.5.1)*.
- Instruction includes using compositions of functions to determine whether two functions are inverses of each other *(MTR.2.1)*.

Common Misconceptions or Errors

- Students may confuse the horizontal line test (used to determine if the function has an inverse) with the vertical line test which determines if the graph represents a function.
- Students may not understand how to transpose the input and output values in a table to determine the inverse of a function.
- Students may not understand that it is possible for a function to have an inverse that may not be a function.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)

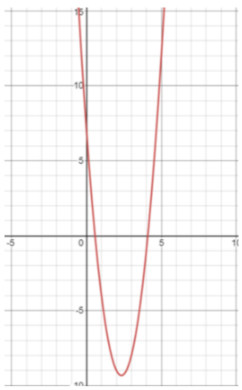
Create a function that has an inverse. Explain using a table, graph, and equation how you know that an inverse exists.

Instructional Task 2 (MTR.5.1)

Create a function for which an inverse does not exist. Explain what about that function makes it not invertible. Explain what adjustments could be made to the function to make it invertible.

Instructional Task 3 (MTR.2.1, MTR.4.1)

Determine whether an inverse function exists for the function given below in the graphical and numerical representations. Explain what features you found in the graph as well as the table that helped you determine whether the function had an inverse or not.



x	$3x^2 - 14x + 7$
-2	47
-1	24
0	7
1	-4
2	-9

Instructional Items

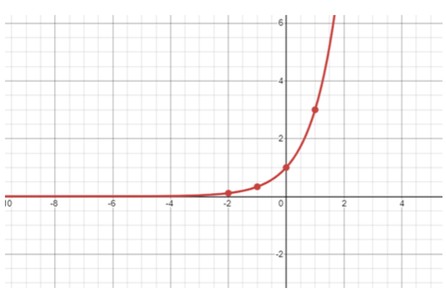
Instructional Item 1

Determine whether an inverse to the following function represented in the table exists. Explain.

x	$2x^2 - 3x - 6$
-2	-4
-1	-7
0	-6
1	-1
2	8

Instructional Item 2

Determine whether an inverse to the following function represented by the following graph exists. Explain.



Instructional Item 3

Determine whether an inverse to the following function represented with the equation exists. Explain.

$$f(x) = \ln(x)$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3.7

Benchmark

MA.912.F.3.7 Represent the inverse of a function algebraically, graphically or in a table. Use composition of functions to verify that one function is the inverse of the other.

Benchmark Clarifications:
Clarification 1: Instruction includes the understanding that a logarithmic function is the inverse of an exponential function.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.6, MA.912.AR.5.7, MA.912.AR.5.8, MA.912.AR.5.9
- MA.912.F.1.6

Terms from the K-12 Glossary

- Inverse functions

Vertical Alignment

Previous Benchmarks

Next Benchmarks

- MA.912.F.3.8, MA.912.F.3.9
- MA.912.C.2.6

Purpose and Instructional Strategies

In Algebra I, students identified and interpreted key features for linear, quadratic, exponential and absolute value functions. In Mathematics for College Algebra, students use those skills around analyzing tables, graphs and equations to represent an inverse to a function given to them *(MTR.2.1, MTR.5.1)*.

- Instruction includes noticing patterns such as in transposing of the input and output values in a table to create an inverse of a function.
- Instruction includes the understanding that an inverse to a function does not always represent a function. Students should be able to know how to look at two functions in any representation and determine if they are inverse functions.
- Instruction includes the connection to MA.912.F.3.6 to help students be able to represent inverses to given functions, if they exist, algebraically, graphically or in a table. If students are not able to determine whether an inverse to a given function exists be examining the features given in the function, they may struggle to be able to represent the inverse of the given function in various representations. In this benchmark, student are also expected to use compositions of functions to verify that one function is the inverse of the other.
- Students need to show understanding that certain function types are inverses of each other such as quadratic, exponential and logarithmic functions. Students should also demonstrate an understanding that a linear function will have a linear inverse, if the

inverse of the function exists *(MTR.5.1)*.

- Instruction includes giving students a function represented in a table, graph, or equation and asking them to determine if the function is invertible (able to produce an inverse function) or non-invertible (not able to produce an inverse function). Once they have determined if the inverse function is able to be produced, then students should be expected to represent that inverse in multiple ways. In a table, students should notice that the input and output values transpose with each other. In a graph, students should notice that the points reflect over the line $y = x$.
- Students should be able to use compositions of function to verity that two functions are inverses of each other. When they compose the functions, they should demonstrate an understanding that if they end up with anything other than x as their solution to the compositions of functions than those functions are not inverses of each other.
- During instruction teachers may ask the students to use another representation (table or graph) to look at the two functions to justify their response as to whether the functions are inverses of each other. If they can see graphically that two functions are inverses of each other but their algebraically they do not get an x when composing the functions, it will prompt the students to go back and check their work. Likewise, if they see graphically or in a table that the two functions are not inverses of each other, but their composition of functions does produce an x they will be prompted to go back and check their algebra *(MTR 6.1)*.
- Instruction includes opportunities for students to explore the idea that when functions are inverses of each other, they can compose the functions both ways [$f(g(x))$ and $g(f(x))$] and still get "x" as the solution. While compositions are not commutative, in this special case, compositions can be done in either order.

Common Misconceptions or Errors

- Students may think that they should get a numerical value (most commonly zero or 1) for their solution when composing the two functions in order for the functions to be inverses, rather than getting the solution "x."
- Students may not understand how the graphs of inverse functions will reflect over the line $y = x$. Students may not realize that it has to reflect over the line $y = x$ rather than just be a reflection over any line.
- Students may not understand how to look for patterns in the table that will justify that a function is an inverse of their given function.

Instructional Tasks

Instructional Task 1(MTR.2.1, MTR.7.1)

The table below shows the number of households in the U.S. in the years.

Year	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**
Households (in thousands)	97,107	98,990	99,627	101,018	102,528	103,874	104,705

Part A. Find a linear function, h, which models the number of households in the U.S. (in thousands) as a function of the year, t.

Part B. Represent the inverse of the linear function you wrote in Part A in a table, graph, and expression.

Instructional Task 2 (MTR.5.1, MTR.4.1)

Let f be the function defined by $f(x) = 10^x$ and g be the function $g(x)$ defined by $f(x) = \log_{10} x$.

a. Sketch the graph of $y = f(g(x))$. Explain your reasoning.
b. Sketch the graph of $y = g(f(x))$. Explain your reasoning.
c. Let f and g be any two inverse functions. For which values of x does $f(g(x)) = x$? For which values of x does $g(f(x)) = x$?

Instructional Task 3 (MTR.2.1, MTR.6.1)

Find the inverse to the function $f(x) = x^2 + 3$.

a. Represent the inverse in an equation/expression, a table, and a graph.
b. Verify that the inverse you found was actually an inverse algebraically using compositions of functions.

Instructional Items

Instructional Item 1

Which of these is **true** for the inverse of the function $f(x) = (x+1)^2$ on the domain $x \leq -1$?

a. $f^{-1}(x) = \sqrt{x} - 1$
b. $f^{-1}(x) = -\sqrt{x} - 1$
c. $f^{-1}(x) = \sqrt{x - 1}$
d. $f^{-1}(x) = -\sqrt{x - 1}$

Instructional Item 2

Find the inverse of the function $f(x) = 3^x$

Instructional Item 3

Find the inverse of the function represented in the table. Represent the inverse in a table.

$\boldsymbol{x}$	$\boldsymbol{2x^2 - 3x - 6}$
-2	-4
-1	-7
0	-6
1	-1
2	8

Instructional Item 4

Verify that the following functions are inverses of each other using compositions of functions.

$$f(x) = 2^x$$
$$g(x) = \log_2 x$$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*