Mathematics for Data and Financial Literacy Honors B.E.S.T. Instructional Guide for Mathematics

The B.E.S.T. Instructional Guide for Mathematics (B1G-M) is intended to assist educators with planning for student learning and instruction aligned to Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards. This guide is designed to aid high-quality instruction through the identification of components that support the learning and teaching of the B.E.S.T. Mathematics Standards and Benchmarks. The B1G-M includes an analysis of information related to the B.E.S.T. Standards for Mathematics within this specific mathematics course, the instructional emphasis and aligned resources. This document is posted on the B.E.S.T. Standards for Mathematics webpage of the Florida Department of Education's website and will continue to undergo edits as needed.

Structural Framework and Intentional Design of the B.E.S.T. Standards for Mathematics

Florida's B.E.S.T. Standards for Mathematics were built on the following.

- The coding scheme for the standards and benchmarks was changed to be consistent with other content areas. The new coding scheme is structured as follows: Content.GradeLevel.Strand.Standard.Benchmark.
- Strands were streamlined to be more consistent throughout.
- The standards and benchmarks were written to be clear and concise to ensure that they are easily understood by all stakeholders.
- The benchmarks were written to allow teachers to meet students' individual skills, knowledge and ability.
- The benchmarks were written to allow students the flexibility to solve problems using a method or strategy that is accurate, generalizable and efficient depending on the content (i.e., the numbers, expressions or equations).
- The benchmarks were written to allow for student discovery (i.e., exploring) of strategies rather than the teaching, naming and assessing of each strategy individually.
- The benchmarks were written to support multiple pathways for success in career and college for students.
- The benchmarks should not be taught in isolation but should be combined purposefully.
- The benchmarks may be addressed at multiple points throughout the year, with the intention of gaining mastery by the end of the year.
- Appropriate progression of content within and across strands was developed for each grade level and across grade levels.
- There is an intentional balance of conceptual understanding and procedural fluency with the application of accurate real-world context intertwined within mathematical concepts for relevance.
- The use of other content areas, like science and the arts, within real-world problems should be accurate, relevant, authentic and reflect grade-level appropriateness.

Components of the B.E.S.T. Instructional Guide for Mathematics

The following table is an example of the layout for each benchmark and includes the defining attributes for each component. It is important to note that instruction should not be limited to the possible connecting benchmarks, related terms, strategies or examples provided. To do so would strip the intention of an educator meeting students' individual skills, knowledge and abilities.

Benchmark
focal point for instruction within lesson or task

This section includes the benchmark as identified in the B.E.S.T. Standards for Mathematics. The benchmark, also referred to as the Benchmark of Focus, is the focal point for student learning and instruction. The benchmark, and its related example(s) and clarification(s), can also be found in the course description. The 9-12 benchmarks may be included in multiple courses; select the example(s) or clarification(s) as appropriate for the identified course.

Connecting Benchmarks/Horizontal Alignment
in other standards within the grade level or course

This section includes a list of connecting benchmarks that relate horizontally to the Benchmark of Focus. Horizontal alignment is the intentional progression of content within a grade level or course linking skills within and across strands. Connecting benchmarks are benchmarks that either make a mathematical connection or include prerequisite skills. The information included in this section is not a comprehensive list, and educators are encouraged to find other connecting benchmarks. Additionally, this list will not include benchmarks from the same standard since benchmarks within the same standard already have an inherent connection.

Terms from the K-12 Glossary

This section includes terms from Appendix C: K-12 Glossary, found within the B.E.S.T. Standards for Mathematics document, which are relevant to the identified Benchmark of Focus. The terms included in this section should not be viewed as a comprehensive vocabulary list, but instead should be considered during instruction or act as a reference for educators.

Vertical Alignment
across grade levels or courses

This section includes a list of related benchmarks that connect vertically to the Benchmark of Focus. Vertical alignment is the intentional progression of content from one year to the next, spanning across multiple grade levels. Benchmarks listed in this section make mathematical connections from prior grade levels or courses in future grade levels or courses within and across strands. If the Benchmark of Focus is a new concept or skill, it may not have any previous benchmarks listed. Likewise, if the Benchmark of Focus is a mathematical skill or concept that is finalized in learning and does not have any direct connection to future grade levels or courses, it may not have any future benchmarks listed. The information included in this section is not a comprehensive list, and educators are encouraged to find other benchmarks within a vertical progression.

Purpose and Instructional Strategies

This section includes further narrative for instruction of the benchmark and vertical alignment. Additionally, this section may also include the following:

- explanations and details for the benchmark;
- vocabulary not provided within Appendix C;
- possible instructional strategies and teaching methods; and
- strategies to embed potentially related Mathematical Thinking and Reasoning Standards (MTRs).

Common Misconceptions or Errors

This section will include common student misconceptions or errors and may include strategies to address the identified misconception or error. Recognition of these misconceptions and errors enables educators to identify them in the classroom and make efforts to correct the misconception or error. This corrective effort in the classroom can also be a form of formative assessment within instruction.

Instructional Tasks

demonstrate the depth of the benchmark and the connection to the related benchmarks

This section will include example instructional tasks, which may be open-ended and are intended to demonstrate the depth of the benchmark. Some instructional tasks include integration of the Mathematical Thinking and Reasoning Standards (MTRs) and related benchmark(s). Enrichment tasks may be included to make connections to benchmarks in later grade levels or courses. Tasks may require extended time, additional materials and collaboration.

Instructional Items

demonstrate the focus of the benchmark

This section will include example instructional items which may be used as evidence to demonstrate the students' understanding of the benchmark. Items may highlight one or more parts of the benchmark.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Mathematical Thinking and Reasoning Standards

MTRs: Because Math Matters

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following.

- The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
- In order to fulfill Florida's unique coding scheme, the 5th place (benchmark) will always be a "1" for the MTRs.
- The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
- At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
- The bulleted language of the MTRs were written for students to use as self-monitoring tools during daily instruction.
- The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
- The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
- The MTRs should not stand alone as a separate focus for instruction, but should be combined purposefully.
- The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

Clarifications:
Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:
Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:
Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

Clarifications:
Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students’ ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:
Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:
Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

Examples of Teacher and Student Moves for the MTRs

Below are examples that demonstrate the embedding of the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction keeping in mind the benchmark(s) that are the focal point of the lesson or task. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.1.1 *Actively participate in effortful learning both individually and collectively.*	• Students engage in the task through individual analysis, student-to-teacher interaction and student-to-student interaction. • Students ask task-appropriate questions to self, the teacher and to other students. *(MTR.4.1)* • Students have a positive productive struggle exhibiting growth mindset, even when making a mistake. • Students stay engaged in the task to a purposeful conclusion while modifying methods, when necessary, in solving a problem through self-analysis and perseverance.	• Teacher provides flexible options (i.e., differentiated, challenging tasks that allow students to actively pursue a solution both individually and in groups) so that all students have the opportunity to access and engage with instruction, as well as demonstrate their learning. • Teacher creates a physical environment that supports a growth mindset and will ensure positive student engagement and collaboration. • Teacher provides constructive, encouraging feedback to students that recognizes their efforts and the value of analysis and revision. • Teacher provides appropriate time for student processing, productive struggle and reflection. • Teacher uses data and questions to focus students on their thinking; help students determine their sources of struggle and to build understanding. • Teacher encourages students to ask appropriate questions of other students and of the teacher including questions that examine accuracy. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.2.1 *Demonstrate understanding by representing problems in multiple ways.*	• Students represent problems concretely using objects, models and manipulatives. • Students represent problems pictorially using drawings, models, tables and graphs. • Students represent problems abstractly using numerical or algebraic expressions and equations. • Students make connections and select among different representations and methods for the same problem, as appropriate to different situations or context. *(MTR.3.1)*	• Teacher provides students with objects, models, manipulatives, appropriate technology and real-world situations. *(MTR.7.1)* • Teacher encourages students to use drawings, models, tables, expressions, equations and graphs to represent problems and solutions. • Teacher questions students about making connections between different representations and methods and challenges students to choose one that is most appropriate to the context. *(MTR.3.1)* • Teacher encourages students to explain their different representations and methods to each other. *(MTR.4.1)* • Teacher provides opportunities for students to choose appropriate methods and to use mathematical technology.
MA.K12.MTR.3.1 *Complete tasks with mathematical fluency.*	• Students complete tasks with flexibility, efficiency and accuracy. • Students use feedback from peers and teachers to reflect on and revise methods used. • Students build confidence through practice in a variety of contexts and problems. *(MTR.1.1)*	• Teacher provides tasks and opportunities to explore and share different methods to solve problems. *(MTR.1.1)* • Teacher provides opportunities for students to choose methods and reflect (i.e., through error analysis, revision, summarizing methods or writing) on the efficiency and accuracy of the method(s) chosen. • Teacher asks questions and gives feedback to focus student thinking to build efficiency of accurate methods. • Teacher offers multiple opportunities to practice generalizable methods.

MTR	Student Moves	Teacher Moves
MA.K12.MTR.4.1 *Engage in discussions that reflect on the mathematical thinking of self and others.*	• Students use content specific language to communicate and justify mathematical ideas and chosen methods. • Students use discussions and reflections to recognize errors and revise their thinking. • Students use discussions to analyze the mathematical thinking of others. • Students identify errors within their own work and then determine possible reasons and potential corrections. • When working in small groups, students recognize errors of their peers and offers suggestions.	• Teacher provides students with opportunities (through open-ended tasks, questions and class structure) to make sense of their thinking. *(MTR.1.1)* • Teacher uses precise mathematical language, both written and abstract, and encourages students to revise their language through discussion. • Teacher creates opportunities for students to discuss and reflect on their choice of methods, their errors and revisions and their justifications. • Teachers select, sequence and present student work to elicit discussion about different methods and representations. *(MTR.2.1, MTR.3.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.5.1 *Use patterns and structure to help understand and connect mathematical concepts.*	• Students identify relevant details in a problem in order to create plans and decompose problems into manageable parts. • Students find similarities and common structures, or patterns, between problems in order to solve related and more complex problems using prior knowledge.	• Teacher asks questions to help students construct relationships between familiar and unfamiliar problems and to transfer this relationship to solve other problems. *(MTR.1.1)* • Teacher provides students opportunities to connect prior and current understanding to new concepts. • Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. *(MTR.3.1, MTR.4.1)* • Teacher allows students to develop an appropriate sequence of steps in solving problems. • Teacher provides opportunities for students to reflect during problem solving to make connections to problems in other contexts, noticing structure and making improvements to their process.
MA.K12.MTR.6.1 *Assess the reasonableness of solutions.*	• Students estimate a solution, including using benchmark quantities in place of the original numbers in a problem. • Students monitor calculations, procedures and intermediate results during the process of solving problems. • Students verify and check if solutions are viable, or reasonable, within the context or situation. *(MTR.7.1)* • Students reflect on the accuracy of their estimations and their solutions.	• Teacher provides opportunities for students to estimate or predict solutions prior to solving. • Teacher encourages students to compare results to estimations and revise if necessary for future situations. *(MTR.5.1)* • Teacher prompts students to self-monitor by continually asking, "Does this solution or intermediate result make sense? How do you know?" • Teacher encourages students to provide explanations and justifications for results to self and others. *(MTR.4.1)*

MTR	Student Moves	Teacher Moves
MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.*	• Students connect mathematical concepts to everyday experiences. • Students use mathematical models and methods to understand, represent and solve real-world problems. • Students investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Students re-design models and methods to improve accuracy or efficiency.	• Teacher provides real-world context to help students build understanding of abstract mathematical ideas. • Teacher encourages students to assess the validity and accuracy of mathematical models and situations in real-world context, and to revise those models if necessary. • Teacher provides opportunities for students to investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them. • Teacher provides opportunities for students to apply concepts to other content areas.

Mathematics for Data and Financial Literacy Honors Area of Emphasis

In Mathematics for Data and Financial Literacy Honors, instructional time will emphasize five areas:

(1) extending knowledge of ratios, proportions and functions to data and financial contexts;
(2) developing understanding of basic economic and accounting principles;
(3) determining advantages and disadvantages of credit accounts and short- and long-term loans;
(4) developing understanding of planning for the future through investments, insurance and retirement plans and
(5) extending knowledge of data analysis to create and evaluate reports and to make predictions.

The purpose of the areas of emphasis is not to guide specific units of learning and instruction, but rather provide insight on major mathematical topics that will be covered within this mathematics course. In addition to its purpose, the areas of emphasis are built on the following.

- Supports the intentional horizontal progression within the strands and across the strands in this grade level or course.
- Student learning and instruction should not focus on the stated areas of emphasis as individual units.
- Areas of emphasis are addressed within standards and benchmarks throughout the course so that students are making connections throughout the school year.
- Some benchmarks can be organized within more than one area.
- Supports the communication of the major mathematical topics to all stakeholders.
- Benchmarks within the areas of emphasis should not be taught within the order in which they appear. To do so would strip the progression of mathematical ideas and miss the opportunity to enhance horizontal progressions within the grade level or course.

The table on the next page shows how the benchmarks within this mathematics course are embedded within the areas of emphasis.

		Ratios, proportions and functions for data and financial contexts	Basic economic and accounting principles	Advantages and Disadvantages of credit accounts and short- and long-term loans	Planning for the future through investments, insurance and retirement plans	Data analysis to create and evaluate reports and to make predictions
Number Sense and Operations	MA.912.NSO.1.1	x		x	x	
	MA.912.NSO.1.2	x		x	x	
	MA.912.NSO.1.6	x		x	x	
	MA.912.NSO.1.7	x		x	x	
Algebraic Reasoning	MA.912.AR.1.1	x	x	x	x	x
	MA.912.AR.1.2	x	x	x	x	x
	MA.912.AR.2.5	x	x			x
	MA.912.AR.3.8	x	x			x
	MA.912.AR.5.7	x		x	x	x
	MA.912.AR.9.10	x	x			
	MA.912.AR.10.1	x		x	x	
	MA.912.AR.10.2	x		x	x	
Functions	MA.912.F.1.2	x	x	x	x	x
	MA.912.F.3.2	x	x			
Financial Literacy	MA.912.FL.1.1	x	x	x	x	x
	MA.912.FL.1.2	x	x			x
	MA.912.FL.1.3	x	x		x	
	MA.912.FL.2.1	x	x	x	x	
	MA.912.FL.2.2		x	x	x	
	MA.912.FL.2.3		x	x	x	
	MA.912.FL.2.4	x	x	x	x	
	MA.912.FL.2.5	x	x	x	x	
	MA.912.FL.2.6	x	x	x	x	
	MA.912.FL.3.1			x	x	
	MA.912.FL.3.2			x	x	

		Ratios, proportions and functions for data and financial contexts	Basic economic and accounting principles	Advantages and Disadvantages of credit accounts and short- and long-term loans	Planning for the future through investments, insurance and retirement plans	Data analysis to create and evaluate reports and to make predictions
	MA.912.FL.3.3			x	x	
	MA.912.FL.3.5			x	x	
	MA.912.FL.3.6			x	x	
	MA.912.FL.3.7			x	x	
	MA.912.FL.3.8			x	x	
	MA.912.FL.3.9			x	x	
	MA.912.FL.3.10			x	x	
	MA.912.FL.3.11			x	x	
	MA.912.FL.4.1			x	x	
	MA.912.FL.4.2			x	x	
	MA.912.FL.4.3			x	x	
	MA.912.FL.4.4			x	x	
	MA.912.FL.4.5			x	x	
	MA.912.FL.4.6			x	x	
Data Analysis & Probability	MA.912.DP.1.2	x	x	x	x	x
	MA.912.DP.2.4	x	x	x		x
	MA.912.DP.2.8	x	x			x
	MA.912.DP.2.9	x	x	x		x
	MA.912.DP.3.1	x				x
	MA.912.DP.3.2	x				x
	MA.912.DP.3.3	x				x
	MA.912.DP.3.4	x				x
	MA.912.DP.5.11	x	x	x	x	x

MA.912.NSO.1 *Generate equivalent expressions and perform operations with expressions involving exponents, radicals or logarithms.*

MA.912.NSO.1.1

Benchmark

MA.912.NSO.1.1	Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.

Benchmark Clarifications:
Clarification 1: Instruction includes the use of technology when appropriate.
Clarification 2: Refer to the K-12 Formulas (Appendix E) for the Laws of Exponents.
Clarification 3: Instruction includes converting between expressions involving rational exponents and expressions involving radicals.
Clarification 4: Within the Mathematics for Data and Financial Literacy course, it is not the expectation to generate equivalent numerical expressions.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1, MA.912.AR.1.2
- MA.912.AR.5.7
- MA.912.AR.10.2
- MA.912.FL.3
- MA.912.FL.4
- MA.912.DP.2.9

Terms from the K-12 Glossary

- Base
- Exponent
- Expression

Vertical Alignment

Previous Benchmarks

- MA.8.NSO.1.3

Next Benchmarks

- Due to multiple pathways in high school, next benchmarks may vary depending on the student.

Purpose and Instructional Strategies

In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rational-number exponents to generate and evaluate numerical expressions (MA.912.NSO.1.1). The focus shifts in Mathematics for Data and Financial Literacy, when students evaluate numerical expressions involving rational exponents that are relevant to financial and data contexts.

- Instruction of this benchmark helps build the foundation for operations with exponents and logarithms throughout the course.
- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.

- Instruction includes a continuation from Algebra I of working with patterns and the connection to mathematical operations and the inverse relationship between powers and radicals (*MTR.5.1*).
- Instruction focuses on a conceptual review of the meaning of integer and fractional exponents.
- Students should make the connection of the root being equivalent to a unit fraction exponent *(MTR.4.1)*.
    - For example, $\sqrt[3]{8} = \sqrt[3]{2^3}$ is equivalent to the equation $\sqrt[3]{8} = (2^3)^{\frac{1}{3}}$, which is equivalent to the equation $\sqrt[3]{8} = 2^{\frac{3}{1}\cdot\frac{1}{3}}$, which is equivalent to the equation $\sqrt[3]{8} = 2^1$, which is equivalent to the equation $\sqrt[3]{8} = 2$.
- Within this courses, bases with rational number exponents are limited to positive values. Students should make the connection that exponential functions are not defined for negative bases, therefore there is little application for raising a negative number to a rational exponent that is not an integer (*MTR.5.1*, *MTR.6.1*, *MTR.7.1*).

Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may try to perform operations on bases as well as exponents.
    - For example, if the expression is $3^2 \cdot 3^3$, a student may rewrite this as 9^5 rather than 3^5.
- Students may multiply the base by the exponent.
- Students may not truly understand exponents that are zero or negative.
- Students may be unsure of their calculations when answers contain multiple decimal places. For simplicity, many exponential problems they have encountered in the past have produced neat, concise solutions. In this course, and in the financial world in general, the numbers are messier. Students need to use their number sense to determine if their solutions are reasonable *(MTR.6.1)* and should expect solutions to often contain multiple decimal places.

Instructional Tasks

Instructional Task 1 (MTR.5.1, MTR.7.1)

Part A. Given the expression $1.011^{12t+500}$, create an equivalent expression in the form of $P\left(1+\frac{r}{12}\right)^{12t}$.

Part B. Write a real-world situation involving money that can be modeled by your expression.

Part C. Evaluate the original expression and the equivalent expression created from Part A for $t = 30$. Interpret the meaning of the result.

Instructional Items

Instructional Item 1

Evaluate the numerical expression $(32)^{\frac{4}{5}}$ using the properties of exponents.

Instructional Item 2

Evaluate the numerical expression $5000\left(1+\frac{0.18}{4}\right)^{(4)\left(\frac{2}{3}\right)}$. Round your answer to the nearest hundredth.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.2

Benchmark

MA.912.NSO.1.2 Generate equivalent algebraic expressions using the properties of exponents.

Example: The expression 1.5^{3t+2} is equivalent to the expression $2.25(1.5)^{3t}$ which is equivalent to $2.25(3.375)^{t}$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1, MA.912.AR.1.2
- MA.912.AR.5.7
- MA.912.AR.10.2
- MA.912.FL.3
- MA.912.FL.4
- MA.912.DP.2.9

Terms from the K-12 Glossary

- Base
- Exponent
- Expression

Vertical Alignment

Previous Benchmarks

- MA.8.AR.1.1

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students generated equivalent algebraic expressions using the Laws of Exponents with integer exponents. In Algebra I, students expanded this work to include rational-number exponents. In Mathematics for Data and Financial Literacy, students generate equivalent algebraic expressions using the properties of exponents and logarithms to solve problems involving money, business or data.

- Instruction of this benchmark helps build the foundation for operations with exponents and logarithms throughout the course.
- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.
- Instruction includes student discovery of the patterns and the connection to mathematical operations (*MTR.5.1*).
  - For example, consider exploring the development of the compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, from MA.912.FL.3.1. Suppose that Jalen invests \$100 in a bank account that pays 5% interest compounded quarterly. If Jalen keeps the

accounts for 15 years, students can determine how much interest is earned and the total value of his investment.

- Students can first explore the interest earned using the simple interest formula, $I = prt$, to find the interest for one quarter. For the first quarter, students should determine that $I = (100)(0.05)\left(\frac{1}{4}\right)$, which is equivalent to $I = (100)\left(\frac{0.05}{4}\right)$.
- The total amount of the investment after one quarter would then be $A_1 = 100 + 100(\frac{0.05}{4})$.

$$A_1 = 100\left(1 + \frac{0.05}{4}\right)$$

- The value of the investment after 2 quarters, A_2, would be

$$A_2 = A_1 + A_1\left(\frac{0.05}{4}\right)$$
$$A_2 = A_1\left(1 + \frac{0.05}{4}\right)$$
$$A_2 = 100\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)$$
$$A_2 = 100\left(1 + \left(\frac{0.05}{4}\right)\right)^2$$

- The value of the investment after 3 quarters, A_3, would be

$$A_3 = A_2 + A_2\left(\frac{0.05}{4}\right)$$
$$A_3 = A_2\left(1 + \frac{0.05}{4}\right)$$
$$A_3 = 100\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)$$
$$A_3 = 100\left(1 + \left(\frac{0.05}{4}\right)\right)^3$$

- The value of the investment after 4 quarters, A_3, would be

$$A_4 = A_3 + A_3\left(\frac{0.05}{4}\right)$$
$$A_4 = A_3\left(1 + \frac{0.05}{4}\right)$$
$$A_4 = 100\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)$$
$$A_4 = 100\left(1 + \left(\frac{0.05}{4}\right)\right)^4$$

At this point, student should see the connection between the exponent 4 and the number of quarters the investment has accumulated interest. Speed the exploration up by moving to 8, 12 and 16 quarters to build the idea of the exponent being the product of the number of years and the number of times the investment compounds each year.

- After 8 quarters (2 years), A_8 would be

$$A_8 = 100\left(1 + \left(\frac{0.05}{4}\right)\right)^8$$

$$A_8 = 100\left(1 + \left(\frac{0.05}{4}\right)\right)^{4(2)}$$

- After 12 quarters (3 years), A_{12} would be

$$A_{12} = 100\left(1 + \left(\frac{0.05}{4}\right)\right)^8$$

$$A_{12} = 100\left(1 + \left(\frac{0.05}{4}\right)\right)^{4(3)}$$

- Students should now see the pattern emerge for the compound interest formula. Have them replace the numbers in the equation to generate $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

- Students should be able to fluently apply the Laws of Exponents in both directions.
  - For example, students should recognize that an annual interest rate of 15%, represented by the expression 1.15^t, can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t}$, which is approximately equivalent to 1.012^{12t}. This latter expression reveals the approximate equivalent monthly interest rate of 1.2%. Similarly, students should recognize the monthly interest rate represented by the expression 1.012^{12t} could be rewritten as $(1.012^{12})^t$, which is approximately equivalent to 1.15^t, which returns to the annual interest rate. Once students see this example, ask them if they could calculate a semi-annual or quarterly interest rate.
    - Note that there is a distinction between the monthly rate that yields an interest rate of 15% per year (which is also known as the annual yearly percentage, or AYP) and the monthly interest rate that is used for an annual percentage rate (APR) of 15% compounded monthly.
- When generating equivalent expressions, students should be encouraged to approach from different entry points and discuss how they are different but equivalent strategies *(MTR.2.1)*.
- The expectation for this benchmark does not include the conversion of an algebraic expression from exponential form to radical form or from radical form to exponential form.

Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may not have fully mastered the Laws of Exponents and understand the mathematical connections between the bases and the exponents.
- Student may believe that with the introduction of variables, the properties of exponents differ from numerical expressions.

Instructional Tasks

Instructional Task 1 (MTR.5.1)
Given the exponential growth function $h(x) = 3^{0.2x}$, calculate the rate of growth.

Instructional Task 2 (MTR.4.1)

Part A. If you invest \$300 and earn \$45 interest for the year, what is the percent interest earned?

Part B. Using the information from Part A, if the interest is being compounded monthly, interpret the equation below. What does r represent?

$$1.15 = (1 + r)^{12}$$

Part C. What is the value of r in Part B?

Part D. Tonya invests \$300 and earns an annual rate of 15% compounded monthly. Explain how the expression below shows how much interests she earns in terms of dollars, assuming that she invests the \$300 for one year.

$$300\left(\left(1 + \frac{15}{12}\right)^{12(1)} - 1\right)$$

Part E. Using the information from Part D, what is the monthly interest rate that Tonya earns?

Part F. Compare the interest you earned (from Parts A to C) to the interest that Tonya earned (from Parts D to E). Explain why the earnings are different.

Instructional Items

Instructional Item 1

Part A. Given the algebraic expression $(2.3)^{2t-1}$, create an equivalent expression.

Part B. Create a financial situation that could be represented by that expression.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.6

Benchmark

MA.912.NSO.1.6 Given a numerical logarithmic expression, evaluate and generate equivalent numerical expressions using the properties of logarithms or exponents.

Benchmark Clarifications:

Clarification 1: Within the Mathematics for Data and Financial Literacy Honors course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.7
- MA.912.FL.3
- MA.912.FL.4

Terms from the K-12 Glossary

- Exponential form
- Inverse function

Vertical Alignment

Previous Benchmarks

- MA.8.NSO.1.3

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rational-number exponents. In Mathematics for Data and Financial Literacy, students extend the Laws of Exponents to the properties of logarithms to evaluate numerical logarithms and generate equivalent numerical logarithmic expressions. Most of the financial formulas students will use in this course are exponential. The exponents in these formulas typically represent a time variable. In scenarios where the length of time for an investment or loan repayment must be calculated, students will need to use logarithms.

- Instruction makes the connection between the properties of logarithms and the properties of exponents. Explain that a logarithm is defined as an exponent establishing the equivalence of $y = a^x$ and, $x = \log_a y$ given $a > 0$ and $a \neq 1$ *(MTR.5.1)*. Remind students that logarithmic and exponential operations are inverse operations, so the properties of the logarithms are the "opposite" of the properties of the exponents.

Properties of Exponents		Properties of Logarithms	
$a^m \cdot a^n = a^{m+n}$	→	$\log_a(m \cdot n) = \log_a m + \log_a n$	Product Property
$\frac{a^m}{a^n} = a^{m-n}$	→	$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$	Quotient Property
$(a^m)^n = a^{mn}$	→	$\log_a(m)^n = n(\log_a m)$	Power Property
		$\log_a(m) = \frac{\log_b m}{\log_b a} = \frac{\log m}{\log a} = \frac{\ln m}{\ln a}$	Change of Base
		Other Properties	
$a^0 = 1$	→	$\log_a 1 = 0$	
$a^1 = a$	→	$\log_a a = 1$	

- Instruction includes making the connection between the change of base formula and the inverse relationship between exponents and logarithms.
  - For example, students should know that by definition $b^{\log_b x} = x$. Therefore, students can take the log with base a of both sides of the equation to obtain $\log_a b^{\log_b x} = \log_a x$. Then, students can use the the Power Property to rewrite the equation as $(\log_b x)(\log_a b) = \log_a x$. Students should notice that there the arguments for two of the logs are b and x with each the same base of a. So, one can divide both sides of the equation by $\log_a b$ to isolate $\log_b x$, obtaining $\frac{(\log_b x)(\log_a b)}{(\log_a b)} = \frac{\log_a x}{(\log_a b)}$. Therefore, $\log_b x = \frac{\log_a x}{\log_a b}$.
- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate them or use the properties *(MTR.4.1)*.
  - For example, present students with $\log_{1.03} 1.092727$ and ask them for its meaning, "the exponent required on the base 1.03 to obtain 1.092727."
- Many examples in this course will require students to utilize the change of base formula to evaluate logarithms. Consider the example above.

$$\log_{1.03} 1.092727 = \frac{\log 1.092727}{\log 1.03} \text{ or } \frac{\ln 1.092727}{\ln 1.03}$$

  - Evaluating this expression with a calculator may require the use of common logs or natural logs. Guide students to understand that common logs have a base of 10 while natural logs have a base of e and are easy to enter into a calculator. Have students use a calculator to evaluate the expression to find it is approximately equivalent to 3. Have students confirm this by checking the exponential equivalent expression $1.03^3 = 1.092727$.

  - While students will see more complex logarithms in this course, it is appropriate to simplify the logarithms they work with initially in this benchmark until they reach an understanding of the properties of logarithms. Depending on the courses students have taken prior to Math for Data and Financial Literacy, this may their first introduction to logarithms.

- Instruction includes the understanding that logarithms with base 10 are called common logarithms and written as $\log x$ or $\log_{10} x$.
- Instruction includes the understanding that logarithms with base e are called natural logarithms and written as $\ln x$ or $\log_e x$.
- Instruction encourages students to read the logarithmic expressions and then discuss the meaning before trying to evaluate it or use the properties.
  - For example, present students with $\log_2 64$ and ask for its meaning, "the exponent required on the base 2 to obtain 64" *(MTR.2.1)*.
- For the purposes of this course, instruction for this benchmark will also include the creation of a logarithmic expression from an exponential expression. Consider the following example: Jaxson deposits $300 into an account that pays 2.3% interest compounded semiannually. If he makes no other deposits, how long will it take for him to have $500 in his account?
  - This problem requires the use of the compound interest formula (discussed further in MA.912.FL.3) where A is the amount the investment is worth, P is the principle, r is the interest rate expressed as a decimal, n is the number of times the interest compounds annually and t is the number of years invested. Substitute the given information and begin to solve.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$500 = 300\left(1 + \frac{.023}{2}\right)^{2t}$$
$$500 = 300(1.0115)^{2t}$$
$$1.\overline{6} = 1.0115^{2t}$$

  - At this point, students should be able to change this into an equivalent logarithmic equation and solve for *t*.

$$2t = \log_{1.0115} 1.\overline{6}$$
$$t = \frac{\log_{1.0115} 1.\overline{6}}{2}$$
$$t = \frac{\log 1.\overline{6}}{\log 1.0115} \div 2 \approx 22.33$$

  - It would take 22.5 years (considering semiannual compounding) for Jaxson's investment to reach $500.
- Problem types include logarithms with different bases, including common logarithms and natural logarithms.

Common Misconceptions or Errors

- Students may see "log" as a variable rather than as an operation.

- For example, they may see "log" as a common factor in the expression, $\log 12 + \log 6$ and mistakenly write $\log(12 + 6)$.
- For example, they may rewrite the expression $\log(12 \cdot 6)$ and as $\log 12 \cdot \log 6$ instead of $\log 12 + \log 6$. Similarly, they may incorrectly rewrite $\log\left(\frac{12}{6}\right)$ as $\frac{\log 12}{\log 6}$ instead of $\log 12 - \log 6$.
- For example, they may cancel the "log" from the numerator and the denominator in an expression.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.7.1)

Rosa deposits $1,000 into an account that pays 3.1% interest compounded monthly. If she makes no other deposits, how long will it take for her to triple her investment?

Instructional Task 2 (MTR.5.1)

Recall that $\log_b(x)$ is by definition the exponent which b must be raised to in order to yield x $(b > 0)$.

Part A.

- Use this definition to compute $\log_2(2^5)$.
- Use this definition to compute $\log_{10}(0.001)$.
- Use this definition to compute $\ln(e^3)$.
- Explain why $\log_b(b^y) = y$ where $b > 0$.

Part B.

- Evaluate $10^{\log_{10}(100)}$.
- Evaluate $2^{\log_2(\sqrt{2})}$.
- Evaluate $e^{\ln(89)}$.
- Explain why $b^{\log_b(x)} = x$ where $b > 0$.

Instructional Task 3 (MTR.7.1)

In 1966, a Miami boy smuggled three Giant African Land Snails into the country. His grandmother eventually released them into the garden, and in seven years there were approximately 18,000 of them. The snails are very destructive and had to be eradicated. According to the United States Department of Agriculture, it took 10 years and cost $1 million to eradicate them.

Part A. Assuming the snail population grows exponentially, write an expression for the population, P, in terms of the number, t, of years since their release.

Part B. How long does it take for the population to double?

Part C. Assuming the cost of eradicating the snails is proportional to the population, how much would it have cost to eradicate them if they had started the eradication program a year earlier? How much would it have cost to eradicate them if they had let the population grow unchecked for another year?

Instructional Items

Instructional Item 1

Which of the following are equivalent to $\log_4 64$?

a. $\log_4 4 + \log_4 16$
b. $\log_4 256 - \log_4 4$
c. $\log_4 8 \cdot \log_4 8$
d. $\frac{\log_4 256}{\log_4 4}$
e. $\frac{\log 64}{\log 4}$
f. $\frac{\ln 4}{\ln 64}$

Instructional Item 2

Using only one of the properties of logarithms, create two equivalent expressions for each of the following. Evaluate each expression to prove their equivalency.

a. $\log 16$
b. $\log_3 81$
c. $\ln 25$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.NSO.1.7

Benchmark

MA.912.NSO.1.7 Given an algebraic logarithmic expression, generate an equivalent algebraic expression using the properties of logarithms or exponents.

Benchmark Clarifications:
Clarification 1: Within the Mathematics for Data and Financial Literacy Honors course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.7
- MA.912.FL.3
- MA.912.FL.4

Terms from the K-12 Glossary

- Exponential function
- Inverse function

Vertical Alignment

Previous Benchmarks

- MA.8.AR.1.1

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students generated equivalent algebraic expressions using the Laws of Exponents with integer exponents. In Algebra I, students worked with rational-number exponents. In Mathematics for Data and Financial Literacy, students generate equivalent algebraic logarithmic expressions. Most of the financial formulas students will use in this course are exponential. The exponents in these formulas typically represent a time variable. In scenarios where the length of time for an investment or loan repayment must be calculated, students will need to use logarithms.

- Be sure students are comfortable and familiar with the properties of logarithms from MA.912.NSO.1.6 prior to instruction on this benchmark *(MTR.3.1)*.

Properties of Exponents		Properties of Logarithms	
$a^m \cdot a^n = a^{m+n}$	→	$\log_a(m \cdot n) = \log_a m + \log_a n$	Product Property
$\frac{a^m}{a^n} = a^{m-n}$	→	$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$	Quotient Property
$(a^m)^n = a^{m \cdot n}$	→	$\log_a(m^n) = n(\log_a m)$	Power Property

		$\log_a m = \frac{\log_b m}{\log_b a} = \frac{\log m}{\log a} = \frac{\ln m}{\ln a}$	Change of Base
		Other Properties	
$a^0 = 1$	$\rightarrow$	$\log_a 1 = 0$	
$a^1 = a$	$\rightarrow$	$\log_a a = 1$	

- Instruction includes making the connection between the change of base formula and the inverse relationship between exponents and logarithms.
  - For example, students should know that by definition $b^{\log_b x} = x$. Therefore, students can take the log with base a of both sides of the equation to obtain $\log_a b^{\log_b x} = \log_a x$. Then, students can use the the Power Property to rewrite the equation as $(\log_b x)(\log_a b) = \log_a x$. Students should notice that there the arguments for two of the logs are b and x with each the same base of a. So, one can divide both sides of the equation by $\log_a b$ to isolate $\log_b x$, obtaining $\frac{(\log_b x)(\log_a b)}{(\log_a b)} = \frac{\log_a x}{(\log_a b)}$. Therefore, $\log_b x = \frac{\log_a x}{\log_a b}$.
- For the purposes of this course, instruction for this benchmark will also include the creation of a logarithmic expression from an exponential expression. Consider the following example *(MTR.5.1)*.
  - Devonte plans to invest some of his money into an investment that compounds annually. As he explores different investments, he wants to know how long it would take for his money to double in each account. Create an expression to calculate the amount of time it would take to double an investment in an account that compounds annually.
    - Begin with the compound interest formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

    - With annual interest, $n = 1$. Doubling Devonte's initial investment would mean $A = 2P$.

$$2P = P(1 + r)^t$$

$$2 = (1 + r)^t$$

    - Have students create an equivalent logarithmic expression to find t.

$$t = \log_{1+r} 2$$

$$t = \frac{\log 2}{\log(1 + r)}$$

  - Have students test this new formula out on a sample investment for multiple interest rates and check their findings with the compound interest formula *(MTR.7.1)*.
    - Once students complete this activity, teach them about the Rule of 72, which states that dividing 72 by the interest rate (given as a percentage) of an investment with a fixed annual rate of interest will approximate the length of time it takes the initial investment to double.

Common Misconceptions or Errors

- Students tend to treat "log" as a variable rather than as an operation:
  - For example, they may see "log" as a common factor in the expression, $\log x + \log y$ and mistakenly write $\log(x+y)$.
  - For example, they may distribute the "log" in the expression $\log(a \cdot b)$ and rewrite it as $\log a \cdot \log b$ instead of $\log_a m + \log_a n$. Similarly they may incorrectly rewrite $\log\left(\frac{a}{b}\right)$ as $\frac{\log a}{\log b}$.
  - For example, hey may divide both sides of the equation $\log(7x-12) = 2\log x$ by "log" to mistakenly obtain $7x - 12 = 2x$.
- Students may cancel the "log" from the numerator and the denominator in an expression. Remind students that just like with roots we can simplify or expand logarithms if the argument is fully factored.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.5.1)

Convert the compound interest formula below, solving it for *t*.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Instructional Task 2 (MTR.7.1)

A hospital is conducting a study to see how different environmental conditions influence the growth of streptococcus pneumonia, one of the bacteria which causes pneumonia. Three different populations are studied giving rise to the following equations:

$$P_1(t) = 1000e^{t/3}$$

$$P_2(t) = 1500e^{3t/8}$$

$$P_3(t) = 5000e^{t/4}$$

Here t represents the number of hours since the beginning of the experiment which lasts for 24 hours and $P_i(t)$ represents the size of the i^{th} bacteria population.

Part A. Explain, in terms of the structure of the expressions defining $P_1(t)$ and $P_2(t)$, why these two populations never share the same value at any time during the experiment.

Part B. Explain, in terms of the structure of the expressions defining $P_1(t)$ and $P_2(t)$, why these two populations will be equal at exactly one time during the experiment. Determine this time.

Instructional Items

Instructional Item 1

Write each expression as a single logarithmic quantity.

i. $\log 7 - \log x$

ii. $3\ln x + 4\ln y - 5\ln z$

iii. $1 + 3\log x$

iv. $\frac{3}{2}\log_2 x^6 - \frac{3}{4}\log_2 x^8$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1 *Interpret and rewrite algebraic expressions and equations in equivalent forms.*

MA.912.AR.1.1

Benchmark

MA.912.AR.1.1 Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.

Algebra I Example: Derrick is using the formula $P = 1000(1 + .1)^t$ to make a prediction about the camel population in Australia. He identifies the growth factor as $(1 + .1)$, or 1.1, and states that the camel population will grow at an annual rate of 10% per year.

Example: The expression 1.15^t can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t}$ which is approximately equivalent to $(1.012)^{12t}$. This latter expression reveals the approximate equivalent monthly interest rate of 1.2% if the annual rate is 15%.

Benchmark Clarifications:
Clarification 1: Parts of an expression include factors, terms, constants, coefficients and variables.
Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.5.7
- MA.912.FL.1
- MA.912.FL.2
- MA.912.FL.3
- MA.912.FL.4

Terms from the K-12 Glossary

- Coefficient
- Equation
- Expression

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.5
- MA.912.AR.2.4
- MA.912.AR.3.7
- MA.912.AR.4.3
- MA.912.AR.5.6

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students interpreted the slope and y-intercept of linear equations within real-world contexts. In Algebra I, students generated and interpreted equivalent linear, absolute value, quadratic and exponential expressions, equations and functions. In Mathematics for Data and Financial Literacy, students work with several finance formulas and identify and interpret components of these formulas in real-world contexts.

- Instruction includes making the connection to linear, quadratic and exponential functions.
- Students should be able to identify factors, terms, constants, coefficients, exponents and variables in expressions and equations.
- Look for opportunities to interpret these components in context – make these discussions part of daily instruction. This benchmark is not intended to be taught in isolation, rather it is intended to create a consistent conversation point across the instruction of other benchmarks.
    - When solving contextual problems (which comprise a large portion of this course), ask students to interpret the parts in different steps of the solution process.
        - For example, if Jamie is calculating the future worth of a $1500 investment with a 2.9% interest rate that compounds annually for 12 years, she can use the compound interest formula below.

$$A = 1500(1 + 0.029)^{12}$$

$A = 1500(1 + 0.029)^{12}$	Principal, Initial amount in the growth factor, Amount of growth per time period, Years of growth
$A = 1500(1.029)^{12}$	Principal, Growth Factor (102.9% of principal each year), Years of growth
$A = 1500(1.40923849245)$	Principal, Total growth Factor (140.9% of initial principal)
$A = 2113.86$	Value of investment after 12 years

Common Misconceptions or Errors

- Students may not be able to identify parts of an expression and equation or interpret those parts within context. Ensure interpretation opportunities are embedded throughout instruction and discussions.
- Students may not see or consider equivalent expressions that could be created through the properties of exponents or logarithms. Prompt students to think in this direction when appropriate.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

There are different formulas used for a periodic investment and a simple investment. For period investments, the formula $A = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}}$ is used to determine the final amount (or balance) at time t. For simple investments, the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ is used to determine the amount at time t. Compare the two formulas using graphing technology. Use different values for P, r, n and t.

Instructional Items

Instructional Item 1

Rashad is calculating the future worth of an investment that compounds annually. He uses the compound interest formula below.

$$A = 3000(1 + 0.032)^{20}$$

Part A. What does the number 3000 represent?
Part B. What does the number 20 represent?

Part C. What does the number 1 represent?
Part D. What does the expression $1 + 0.032$ represent?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.1.2

Benchmark

MA.912.AR.1.2 Rearrange equations or formulas to isolate a quantity of interest.

Algebra I Example: The Ideal Gas Law $PV = nRT$ can be rearranged as $T = \frac{PV}{nR}$ to isolate temperature as the quantity of interest.
Example: Given the Compound Interest formula $= P\,(1 + \frac{r}{n})^{nt}$, solve for P.
Mathematics for Data and Financial Literacy Honors Example: Given the Compound Interest formula $A = P(1 + \frac{r}{n})^{nt}$, solve for t.

Benchmark Clarifications:
Clarification 1: Instruction includes using formulas for temperature, perimeter, area and volume; using equations for linear (standard, slope-intercept and point-slope forms) and quadratic (standard, factored and vertex forms) functions.
Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.2, MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.5.7
- MA.912.FL.1
- MA.912.FL.2
- MA.912.FL.3
- MA.912.FL.4

Terms from the K-12 Glossary

- Equation

Vertical Alignment

Previous Benchmarks

- MA.8.AR.2.3
- MA.912.AR.2.1
- MA.912.AR.3.1
- MA.912.AR.4.1

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students isolated variables in equations in the form $x^2 = p$ and $x^3 = q$. In Algebra I, students isolated a variable or quantity of interest in linear, absolute value and quadratic equations and formulas. In this course, students isolate a variable or quantity of interest for multiple formulas, including exponential and logarithmic equations.

- Students should understand that in financial contexts, multiple variables exist. There will be times when students may want to focus on the value of any one of these variables, which results in the need to solve a given formula for that variable. Rearranging formulas to isolate the variable can make that exploration much easier for repeated calculations, especially when using spreadsheet technology.

- Instruction includes justifying each step while rearranging an equation or formula.
    - For example, when rearranging $A = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}}$ for P, it may be helpful for students to highlight the quantity of interest with a highlighter, so students remain focused on that quantity for isolation purposes. Additionally, it may be helpful for students to identify and shift parts of the equations as expressions instead of attempting to shift individual variables or numbers (MA.912.AR.1.1).

$A = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}}$ Start with the original formula.

$A\left(\frac{r}{n}\right) = P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)$ Multiply both sides by $\frac{r}{n}$.

$P = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt}-1}$ Divide both sides by $\left(1+\frac{r}{n}\right)^{nt}-1$.

Common Misconceptions or Errors

- Students may not have mastered the inverse arithmetic operations.
- Students may not see that inverse operations can be performed with groups of numbers and variables treated as a single part of the formula (MA.912.AR.1.1).
- Students may be frustrated because they are not arriving at a numerical value as their solution. Remind students that they are rearranging variables that can be later evaluated as a numerical value.
- Having multiple variables and no values may confuse students and make it difficult for them to see the connections between rearranging a formula and solving one-variable equations.

Instructional Tasks

Instructional Task 1 (MTR.4.1)

The formula below calculates the present value of a single deposit investment.

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Part A. Solve the formula for r.
Part B. Why would it be helpful to have the formula rearranged to isolate r?
Part C. Solve the formula for t.
Part D. Why would it be helpful to have the formula rearranged to isolate t?

Instructional Items

Instructional Item 1

Solve the continuous compound interest formula $A = Pe^{rt}$ for t.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.2 *Write, solve and graph linear equations, functions and inequalities in one and two variables.*

MA.912.AR.2.5

Benchmark

MA.912.AR.2.5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.

Algebra I Example: Lizzy's mother uses the function $C(p) = 450 + 7.75p$, where $C(p)$ represents the total cost of a rental space and p is the number of people attending, to help budget Lizzy's 16th birthday party. Lizzy's mom wants to spend no more than $850 for the party. Graph the function in terms of the context.

Benchmark Clarifications:
Clarification 1: Key features are limited to domain, range, intercepts and rate of change.
Clarification 2: Instruction includes the use of standard form, slope-intercept form and point-slope form.
Clarification 3: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.
Clarification 4: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.
Clarification 5: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1, MA.912.AR.1.2
- MA.912.AR.9.10
- MA.912.AR.10.1
- MA.912.F.1.2
- MA.912.F.3.2
- MA.912.FL.2.6
- MA.912.FL.3.1, MA.912.FL.3.2
- MA.912.DP.2.4, MA.912.DP.2.8, MA.912.DP.2.9

Terms from the K-12 Glossary

- Coordinate
- Domain
- Function
- Function notation
- Linear function
- Piecewise function
- Range
- Rate of change
- Set-builder notation
- Slope
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.8.AR.3.4, MA.8.AR.3.5
- MA.912.F.1.5, MA.912.F.1.6, MA.912.F.1.8

Next Benchmarks

Purpose and Instructional Strategies

In grade 8, students determined and interpreted the slope and y-intercept of a two-variable linear equation in slope-intercept form from a real-world context. In Algebra I, students solved real-world problems that are modeled with linear functions when given equations in all forms, and they determined and interpreted the domain, range and other key features. In Mathematics for Data and Financial Literacy, students focus previous learning on money and business problems (*MTR.7.1*).

- Instruction includes a variety of real-world contexts like the ones described below.
    - Students should understand how cars appreciate or depreciate over time. The simplest form is straight line depreciation.
    - Calculating commission rates for salespeople.
    - Expressing net proceeds when purchasing and selling stocks.
    - Calculating fixed and variable expenses.
- Within the real-world context, instruction includes the use of x-y notation and function notation.
- When specific contexts are modeled by linear functions, parts of the domain and range may not make sense and need to be removed, creating the need for constraints.
    - For example, when a salesperson makes a sale their commission is based on the amount of the sale.
- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation.
- Words

  If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
- Inequality notation

  If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation

      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation

      If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- Depending on a student's pathway, they may not have worked with interval notation (as that was not an expectation in Algebra I) before this course. Instruction includes making connections between inequality notation and interval notation.
    - For example, if the range of a function is $-10 < y < 24$, it can be represented in interval notation as $(-10, 24)$. This is commonly referred to as an open interval because the interval does not contain the end values.
    - For example, if the domain of a function is $0 \leq x \leq 11.5$, it can be represented in interval notation as $[0, 11.5]$. This is commonly referred to as a closed interval because the interval contains both end values.
    - For example, if the domain of a function is $0 \leq x < 50$, it can be represented in interval notation as $[0, 50)$. This is commonly referred to as a half-open, or half-closed, interval because the interval contains only one of the end values.

- For example, if the range of a function is all real numbers, is can be represented in interval notation as $(-\infty, \infty)$. This is commonly referred to as an infinite interval because at least one of end values is infinity (positive or negative).

- Instruction includes the understanding that both discrete and continuous linear contexts can be represented by a linear two-variable equation.
  - Be sure to review the idea of discrete versus continuous data with students and guide them to discuss its impact on making sense of solutions (*MTR.4.1*).
- For mastery of this benchmark, students should be given flexibility to represent discrete linear contexts as either a line or a set of points.
- Instruction includes the use of technology to develop the understanding of constraints.

Common Misconceptions or Errors

- Students may assign their constraints to the incorrect variable.
- Students may misunderstand representing interval notation from smallest to largest when representing the domain or range. To address this misconception, have students sketch the graph to demonstrate their understanding of representing the domain from left to right and range from bottom to top of their graph.
- Students may confuse an open interval with an ordered pair.
- Students may miss the need for compound inequalities in their constraints. Students may not include zero as part of the domain or range.

Instructional Tasks

Instructional Task 1 (MTR.4.1)

Ron works at a technology store and earns commission based on his total sales each day.

- If the total sales is less than $250, the commission is 15%.
- If the total sales between $250 and $550, the commission is 15% of the first $250 plus 20% of the amount over $250.
- If the total sales is more than $550, the commission is 15% of first $250 plus 20% of the next $300 plus 30% of the amount over $550.

If x represents the profit, graph a function to represent the commission, $c(x)$.

Instructional Task 2 (MTR.5.1)

Fiona purchases a new car for $26,400. She gave a $3,000 down payment and pays $422 per month for 60 months. She did some research and found that her car will straight line depreciate to $500 after 20 years.

Part A. Create an expense and a depreciation function based on the situation.

Part B. Interpret the region before, at, and after the intersection point.

Instructional Items

Instructional Item 1

Reese purchased a new car for $36,450. The car straight line depreciates to $600 after 22 years.

Part A. Identify the coordinates of the x- and y-intercepts for the depreciation equation.

Part B. Determine the slope of the depreciation equation.

Part C. Write the straight-line depreciation equation that models the situation.

Part D. State the domain, range and any constraints.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.3 *Write, solve and graph quadratic equations, functions and inequalities in one and two variables.*

MA.912.AR.3.8

Benchmark

MA.912.AR.3.8 Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.

Algebra I Example: The value of a classic car produced in 1972 can be modeled by the function $V(t) = 19.25t^2 - 440t + 3500$, where t is the number of years since 1972. In what year does the car's value start to increase?

Benchmark Clarifications:
Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.
Clarification 2: Instruction includes the use of standard form, factored form and vertex form.
Clarification 3: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.
Clarification 4: Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1, MA.912.AR.1.2
- MA.912.AR.2.5
- MA.912.AR.9.10
- MA.912.F.1.2
- MA.912.F.3.2
- MA.912.DP.2.4, MA.912.DP.2.8

Terms from the K-12 Glossary

- Coordinate Plan
- Domain
- Function
- Function notation
- Quadratic expression
- Quadratic function
- Range
- Set-builder notation
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.3.1, MA.912.AR.3.4, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students were introduced to quadratic functions and learned to wrote, graph and interpret their key features. In Math for Data and Financial Literacy, students continue their work with quadratic functions focusing on data and financial contexts (*MTR.7.1*).

- Instruction focuses on real-world contexts that require students to create a function as a tool to determine requested information or should provide the group of functions that models the context.
  - For example, students can use quadratics to find the revenue, which is the total

amount a company collects from the sale of a product or service. Revenue (R) can be found by the product of the purchase price (p) and quantity (q), $R = pq$. Students should recognize that depending on the type of functions used to model price and quantity, the revenue may be modeled by a quadratic function.

- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when solving the quadratic in one form might be more useful than other depending on context.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
  - Inequality notation
    If the domain is all values of x greater than 2, it can be represented as $x > 2$.
  - Set-builder notation
    If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
  - Interval notation
    If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- In conversations with students, prompt them to reflect on what they know about the context and how they can use that information to determine the requested information.

Common Misconceptions or Errors

- Students may assign their constraints to the incorrect variable.
- Students may miss the need for compound inequalities in their constraints. Students may not include zero as part of the domain or range.

Instructional Tasks

Instructional Task 1 (MTR.4.1)

An item in Ely's shop costs \$8.50 to manufacture. The fixed costs are \$15,000 with a demand function is $q = -250p + 20{,}000$, where q is the quantity the public will buy given the price p.

Part A. Write the revenue function for the item from Ely's Shop.
Part B. Compare your revenue with a partner.
Part C. Draw and label the graph of the expense function.
Part D. Graph the revenue function in a suitable viewing window.
Part E. What price will yield the maximum revenue? What is the revenue at that price? Round answers to the nearest cent.

Instructional Items

Instructional Item 1

A new coffee shop wants to maximize their profit within the first year of business. They determined the function, $P(x) = -x^2 + 6x + 10$, models the profit they can earn in tens of thousands of dollars in terms of the price per cup of coffee, in dollars.

Part A. Transform the function to determine and interpret its vertex.

Part B. Transform the function to determine and interpret its intercepts.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.5 *Write, solve, and graph exponential and logarithmic equations and functions in one and two variables.*

MA.912.AR.5.7

Benchmark

MA.912.AR.5.7 Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context.

Example: The graph of the function $f(t) = e^{5t+2}$ can be transformed into the straight line $y = 5t + 2$ by taking the natural logarithm of the function's outputs.

Benchmark Clarifications:

Clarification 1: Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior and asymptotes.

Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Clarification 3: Instruction includes understanding that when the logarithm of the dependent variable is taken and graphed, the exponential function will be transformed into a linear function.

Clarification 4: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.2, MA.912.NSO.1.7
- MA.912.AR.1.1, MA.912.AR.1.2
- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.9.10
- MA.912.AR.10.2
- MA.912.F.1.2
- MA.912.F.3.2
- MA.912.FL.3
- MA.912.FL.4

Terms from the K-12 Glossary

- Domain
- Exponent
- Exponential function
- Function
- Set-builder notation

Vertical Alignment

Previous Benchmarks

- MA.912.AR.5.3, MA.912.AR.5.4, MA.AR.5.6

Next Benchmarks

Purpose and Instructional Strategies

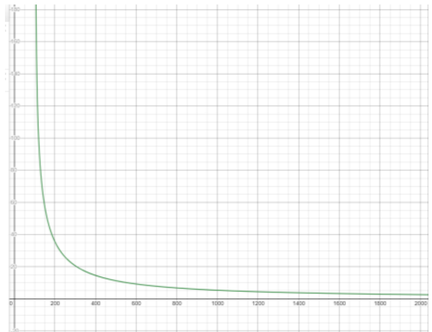
In Algebra I, students worked with exponential functions in limited forms. In Math for Data and Financial Literacy, students solve problems modeled with exponential functions.

- Instruction focuses on real-world contexts that require students to create a function as a tool to determine requested information or should provide the group a function that models the context.
    - For example, students can use exponential decay when analyzing the historical depreciation of a car. In MA.912.AR.2.5, students find the straight-line depreciation. In this benchmark, students could find the percentage the car depreciates instead of the dollar amount (exponential depreciation) using the equation $y = A(1 - r)^x$, where A is the starting value of the car, r is the percent of depreciation, x is the elapsed time in years, and y is the value of the car after x years.
- Instruction includes making the connection to logarithms when working with investments formulas involving exponents.
    - For example, when using the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where r is the interest rate expressed as a decimal, n is the number of times the interest is compounded annually, and t is the term of the account, students may need to use logarithms to find the value of a variable.
    - For example, when using the formula $A = Pe^{rt}$, students will need to use the natural logarithm to isolate the variables r and t.

$$A = Pe^{rt}$$
$$\frac{A}{P} = e^{rt}$$
$$\ln\frac{A}{P} = \ln e^{rt}$$
$$\ln\frac{A}{P} = rt$$
$$t = \frac{\ln\left(\frac{A}{P}\right)}{r}$$

- Instruction includes the use of technology and various representations of functions when solving problems. Given the complexity of the formulas in this course, it may be helpful for students to solve certain problems graphically rather than algebraically. Students can explore entering equations using graphing software.
    - For example, if Sue deposited \$5,000 into an account that compounds interest monthly with an annual rate (APR) of 1.5%, students can determine how long it will take until the account reaches \$30,000 algebraically or graphically. To determine this graphically, students can graph the function $y = 5000\left(1 + \frac{0.015}{12}\right)^{12t}$ using technology. Students should realize that by finding where the graph reaches the y-value of 30,000, the corresponding t-value will provide how long it would take.
    - For example, students can graph the equation $20000 = W\frac{1-\left(1+\frac{0.021}{4}\right)^{-4t}}{\frac{0.021}{4}}$ to

explore the relationship between the withdrawal amount and the time it takes to fully deplete a $20,000 balance from an account that compounds quarterly at 2.1%.



- Problem types include using instances when periodic deposits are made into an account, and using formulas to find the term of systematic savings account. The formula to calculate the future value of a periodic deposit investment is $B = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}}$, and the formula for the present value of a periodic deposit investment is $P = \frac{B \times \frac{r}{n}}{\left(1+\frac{r}{n}\right)^{nt}-1}$.
- Problem types include using formulas to find the term of systematic withdrawals, which is when withdrawals are made at regular intervals. The formula to figure out the present value is $P = W\,\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$.
- Be sure to highlight the relative ease and usefulness of graphing financial formulas to explore relationships. Graphs provide the visual data from an infinite number of calculations, saving students time and hassle when exploring relationships between variables.
- Instruction includes making connections to various forms of exponential functions to show their equivalency. Students should understand and interpret when solving the exponential in one form might be more useful than solving in another form depending on context.
- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and intervals where the function is increasing, decreasing, positive or negative, using words, inequality notation, set-builder notation and interval notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation
      If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as

$(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.

Common Misconceptions or Errors

- Ensure students are plugging the correct information into the formulas and using technology correctly. Grouping symbols on graphic software can be tricky. Model the entry of a few formulas with grouping symbols for students before releasing them to explore their own.
- Students may confuse finding the logarithm and natural logarithm.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1)

Luisa deposited $15,000 into a savings account, which compounds interest monthly with an annual rate (APR) of 1.15%. Each month, she withdraws $350 from this account to pay for travel expenses.

Part A. How long will it take until the account is fully depleted?

Part B. How can she change the amount she withdrawals so her account lasts twice as long?

Instructional Task 2 (MTR.7.1)

Han wants to set up a periodic investment account to save for a future car purchase. His goal is to save $40,000. The best account he's found pays a 2.3% annual interest rate, compounded monthly. Use the periodic investment formula below to graph a relationship between the amount of his monthly periodic investment, P, and the time it will take to achieve his goal, t.

$$A = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\frac{r}{n}}$$

Instructional Items

Instructional Item 1

Mindy purchased a 3-year-old car for $22,300. When the car was new, it sold for $35,500. Find the depreciation rate to the nearest hundredth of a percent.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.9 *Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.*

MA.912.AR.9.10

Benchmark

MA.912.AR.9.10 Solve and graph mathematical and real-world problems that are modeled with piecewise functions. Interpret key features and determine constraints in terms of the context.

Example: A mechanic wants to place an ad in his local newspaper. The cost, in dollars, of an ad x inches long is given by the following piecewise function. Find the cost of an ad that would be 16 inches long.

$$C(x) = \begin{Bmatrix} 12x & x < 5 \\ 60 + 8(x - 5) & x \geq 5 \end{Bmatrix}$$

Benchmark Clarifications:
Clarification 1: Key features are limited to domain, range, intercepts, asymptotes and end behavior.
Clarification 2: Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.5.7
- MA.912.AR.10.1, MA.912.AR.10.2
- MA.912.F.1.2
- MA.912.FL.2.6

Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Function
- Function notation
- Piecewise function
- Range (of a relation or a function)
- Rate of change
- Set-builder notation
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.4
- MA.912.AR.3.7
- MA.912.AR.4.3
- MA.912.AR.5.6

Next Benchmarks

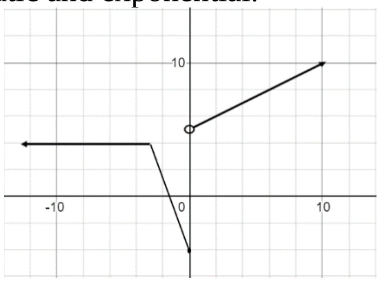
Purpose and Instructional Strategies

In Algebra I, students worked with linear, quadratic, absolute value and exponential functions. In Math for Data and Financial Literacy, students work with piecewise functions, interpret the key features of their graphs, and determine constraints in terms of the context.

- Instruction includes real-world problems that can be modeled with piecewise functions *(MTR.7.1)*.
- It is important for students to understand that a piecewise function is a function defined

by multiple sub functions, each of which applies to a certain interval defined by the function's domain.

- For mastery of this benchmark within this course, sub functions are limited to linear, quadratic and exponential.



$$f(x) = \begin{cases} 4, & x < -3 \\ -\frac{8}{3}x - 4, & -3 \leq x < 0 \\ \frac{1}{2}x + 5, & x > 0 \end{cases}$$

- Instruction includes the use of x-y notation and function notation.
- Instruction includes representing domain, range and any constraints using words, inequality notation, set-builder notation and interval notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation
      If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- This benchmark expects students to interpret key features and determine constraints in terms of the context of a real-world problem *(MTR.7.1)*.
    - For the example used in the benchmark, the piecewise function models the cost, in dollars, of an ad x inches long. The key features and constraints are as follows.
        - The domain of a function is the complete set of possible values of the input of a function or relation. The input values in this case are the size in inches of the ad length. The minimum ad length someone could possibly have is 0 inches. The maximum ad length (based on the length of traditional newspapers in the United States) would be the length of a full page, which is approximately 21 inches. The set of inputs could be considered continuous within the minimum and maximum since the length is not limited to only whole numbers. Therefore, a reasonable domain for this context would be any length from 0 to 21 inches.
        - The range of a function is the complete set of possible values of the output of a function or relation. In this case, the output is the cost of the ad. The minimum cost of an ad would be 0 if the ad size length is 0. This function uses the subset function $12x$ when $x < 5$. The upper bound of the cost for this subset would be $12(5)$, or 60, but does not include 60. Once the size of the ad reaches 5 inches in length, the subset function $60 + 8(x - 5)$ is

used. The lowest cost for this subset would be $60 + 8(5 - 5) = 60$ and the highest cost for that subset would be $60 + 8(21 - 5) = 188$. Overall, the lowest cost would be 0 and the highest cost would be 188, so the range is all values between 0 and 188, inclusive.

- The x-intercept occurs when the output value is 0. The y-intercept occurs when the input value is 0. In this case, the x-intercept occurs when the cost of the ad is equal to 0. The cost of the ad is 0 when the length of the ad is 0 inches. The $C(x)$-intercept is the starting cost of the ad when the ad length is 0. The starting cost of the ad when the ad length is 0 is 0 dollars. Therefore, the x-intercept and the $C(x)$-intercept is the origin (0,0).

Common Misconceptions or Errors

- When evaluating a piecewise function for a specific input, students may attempt to substitute the input value into the wrong subset function or may try to substitute the input value into all the subset functions.
- When graphing piecewise functions, students may forget to put an open circle at the endpoint of a subset function when the endpoint is not included in the domain of the subset function.
- Students may have trouble interpreting the range when the range is not continuous and joins two or more intervals.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

The function below models the height, $h(x)$, in inches, of water in an aquarium x minutes after beginning to fill the aquarium.

$$h(x) = \begin{cases} 2x, & x < 2 \\ x + 2, & 2 \leq x < 5 \\ 7, & 5 \leq x \leq 21 \end{cases}$$

Part A. Graph the function. Be sure to label your graph.
Part B. Write a story to describe what is happening in your graph.
Part C. Interpret the domain, range and intercepts of the function in the context of the problem.
Part D. What will the height of the water be 2 minutes after beginning to fill the aquarium?

Instructional Items

Instructional Item 1

Miami airport offers several different parking options for visitors. A new short term (no more than 24 hours) parking garage charges $1 per hour (or any part thereof) for the first two hours and after that, $3 per hour (or any part thereof) not to exceed $20 per day.

Part A. Graph the function described above.
Part B. Interpret the domain and range of the function in the context of the problem.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.10 *Solve problems involving sequences and series.*

MA.912.AR.10.1

Benchmark

MA.912.AR.10.1 Given a mathematical or real-world context, write and solve problems involving arithmetic sequences.

Example: Tara is saving money to move out of her parent's house. She opens the account with $250 and puts $100 into a savings account every month after that. Write the total amount of money she has in her account after each month as a sequence. In how many months will she have at least $3,000?

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.9.10
- MA.912.FL.3

Terms from the K-12 Glossary

- Arithmetic sequence

Vertical Alignment

Previous Benchmarks

- MA.912.AR.2.1, MA.912.AR.2.2

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students wrote and solved equations of linear functions. In Math for Data and Financial Literacy, students write and solve problems using arithmetic sequences.

- Instruction focuses on real-world contexts involving arithmetic sequences *(MTR.7.1)*.
- Students may use a variety of representations to solve problems such as writing out the sequence using the explicit formula. The focus of the benchmark is to understand arithmetic sequences and how they relate to linear equations *(MTR.2.1)*.
    - In the example given in the benchmark, students are asked to write the total amount of money she has in her account after each month as a sequence. In how many months will she have at least $3,000? The sequence goes as follows: 350, 450, 550, 650, 750, 850, 950, 1050, 1150, 1250, 1350, 1450, 1550, 1650, 1750, 1850, 1950, 2050, 2150, 2250, 2350, 2450, 2550, 2650, 2750, 2850, 2950, 3050, …
      Once students have written the sequence, they can simply count the number of months it takes to get to at least $3,000. It would take 28 months.
    - Another method to solve this would be to use the explicit formula. In the case of using the explicit formula, students would not need to write as many terms in the sequence since solving the problem this way would not involve counting all the terms until getting to $3,000. When using the explicit formula, arithmetic sequences have a linear relationship where the constant change is the slope while the y-intercept is the starting amount. Students can use the equation $y = 100x + 250$, where x is the number of months and y is the amount of money in her account. To solve this problem students would substitute $3,000 with the y-value.

$$3000 = 100x + 250$$
$$2750 = 100x$$

$$27.5 = x$$

  - Since she is only putting money in once per month, the x would need to be a whole number. She wouldn't quite have $3,000 after 27 months so the solution would need to be rounded up to 28 months *(MTR.6.1)*.
- If the y-intercept is unknown, students can use point-slope form of a line and any term in the sequence to write the formula for the sequence. For example, in the second month she has $450. Using point-slope form of a line, students get $y - 450 = 100(x - 2)$.

$$y - 450 = 100(x - 2)$$
$$3000 - 450 = 100x - 200$$
$$2550 = 100x - 200$$
$$2750 = 100x$$
$$27.5 = x$$

Common Misconceptions or Errors

- Students may have a hard time distinguishing between arithmetic sequences and geometric sequences.
- Students may have trouble using the equation by confusing the slope and y-intercept or not knowing to use point-slope form when the y-intercept is unknown.
- Students may not round correctly when needed depending on the context.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.7.1)

Albuquerque boasts one of the longest aerial trams in the world. The tram transports people up to Sandia Peak. The table shows the elevation of the tram at various times during a particular ride. The tram increases elevation by 255 feet every minute. After 1 minute, the elevation of the tram is 6,814 feet.

Part A. What was the elevation at the beginning of the ride?

Part B. If the ride took 15 minutes, what was the elevation of the tram at the end of the ride?

Instructional Items

Instructional Item 1

In a theater, there are 16 seats in the first row and the number of seats in each row after that increase by 2 successively. There are 35 rows in the theater. How many seats does the last row in the theater have?

a. 65
b. 78
c. 84
d. 86

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.AR.10.2

Benchmark

MA.912.AR.10.2 Given a mathematical or real-world context, write and solve problems involving geometric sequences.

Example: A bacteria in a Petri dish initially covers 2 square centimeters. The bacteria grows at a rate of 2.6% every day. Determine the geometric sequence that describes the area covered by the bacteria after 0, 1, 2, 3 … days. Determine, using technology, how many days it would take the bacteria to cover 10 square centimeters.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.7
- MA.912.AR.9.10

Terms from the K-12 Glossary

- Geometric sequence

Vertical Alignment

Previous Benchmarks

- MA.912.AR.5.4

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students worked with exponential relationships. In Math for Data and Financial Literacy, students write and solve problems using geometric sequences.

- Instruction focuses on real-world contexts involving geometric sequences *(MTR.7.1)*.
- Students may use a variety of representations to solve problems such as writing out the sequence or using the explicit formula. The focus of the benchmark is to understand geometric sequences and how they connect to exponential equations *(MTR.2.1)*.
- In cases where the student is solving for time, students can use logarithms, or can use technology, such as an online or handheld graphing calculator, to solve with a graph, table or substitution.

Common Misconceptions or Errors

- Students may mistake a geometric model for an arithmetic model.
- Students may forget to convert the percent to a decimal when performing calculations.
- Students may forget to add 1 to the rate when a geometric sequence is growing or to subtract the rate from 1 when the sequence models a decay model.
- Students may have trouble using technology to determine a value when solving for time.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.5.1, MTR.7.1)

In January, Juan put $1200 in a savings account that pays 3.0% interest once at the end of each year. Assuming Juan has no other deposits or withdrawals, answer the following.

Part A. Determine the geometric sequence that describes the amount of money in his account after 0, 1, 2, 3… years.

Part B. How long will it take for Juan to double his money?

Instructional Items

Instructional Item 1

A company starts a new website. On the first day of the website being published, the site has 250 visitors. The company estimates that the number of visits to the website will increase 5% each day.

Part A. Write a sequence that describes the number of visits on each of the first 5 days.
Part B. How long will it take for the website to have 1,000 visits in a single day?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.1 *Understand, compare and analyze properties of functions.*

MA.912.F.1.2

Benchmark

MA.912.F.1.2 Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Algebra I Example: The function $f(x) = \frac{x}{7} - 8$ models Alicia's position in miles relative to a water stand x minutes into a marathon. Evaluate and interpret for a quarter of an hour into the race.

Benchmark Clarifications:
Clarification 1: Problems include simple functions in two-variables, such as $f(x, y) = 3x - 2y$.
Clarification 2: Within the Algebra I course, functions are limited to one-variable such as $f(x) = 3x$.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.5.7
- MA.912.AR.9.10
- MA.912.F.3.2

Terms from the K-12 Glossary

- Domain
- Function notation

Vertical Alignment

Previous Benchmarks

- MA.6.AR.1.3

Next Benchmarks

Purpose and Instructional Strategies

In middle grades, students worked with x-y notation and substituted values in expressions and equations. In Algebra I, students work with x-y notation and function notation throughout instruction of linear, quadratic, exponential and absolute value functions. In Math for Data and Financial Literacy, students continue to use function notation focusing on functions described within financial and data contexts.

- Instruction leads students to understand that $f(x)$ reads as "f of x" and represents an output of a function in the same way that the variable y represents the output in x-y notation.
- Instruction includes a series of functions with variety of inputs so that students can see the pattern that emerges *(MTR.5.1)*.
    - For example, students can start with $f(x) = 2x^2 + 5x - 7$ and then substitute -2 for x, denoted by $f(-2) = 2(-2)^2 + 5(-2) - 7$. Then, students can substitute a different variable for x, like k, and denote it by $f(k) = 2k^2 + 5k - 7$.
- Students should discover that the number in parenthesis corresponds to the input or x-value on the graph and the number on the other side of the equal sign corresponds to the

output or y-value.

- Although not conventional, instruction includes using function notation flexibly.
    - For example, function notation can been seen as $h(x) = 4x + 7$ or $4x + 7 = h(x)$.
- Instruction leads students to consider the practicality that function notation presents to mathematicians. In several contexts, multiple functions can exist that we want to consider simultaneously. If each of these functions is written in x-y notation, it can lead to confusion in discussions.
    - For example, the equations $y = -2x + 4$ and $y = 3x + 7$. Representing these functions in function notation allows mathematicians to distinguish them from each other more easily (i.e., $f(x) = -2x + 4$ and $g(x) = 3x + 7$).
- Function notation allows for the use of different symbols for the variables, which can add meaning to the function.
- A function of two variables has inputs that are ordered pairs (x, y) and the outputs are a single real number. The domain of the function is a set of the ordered pairs and the range is the set of all possible outputs.
    - For example, if you rent a car, the cost depends on two items, the days you keep the car and how far you drive. As a function this would be represented as the cost of the car rental, R, with the inputs of the function being the days driven, d, and the miles driven, m. The function in two variables would be represented as $R(d, m)$ in function notation.

Common Misconceptions or Errors

- Throughout students' prior experience, two variables written next to one another indicate they are being multiplied. That changes in function notation and will likely cause confusion for some of your students. Continue to discuss the meaning of function notation with these students until they become comfortable with the understanding. In other words, $f(x)$ does not mean $(f) \times (x)$.
- Students may need additional support in the order of operations while working with functions.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.5.1)

The rental cost in dollars for renting a car for n days and driving it m miles is represented by the equation $R(n, m) = 30n + 0.35m$.

Part A. What is the cost of renting the car for 5 days and driving it for 100 miles?

Part B. What is the result of $R(200, 30)$? What does the result mean in terms of renting the car?

Part C. If you rent the car for 4 days, and only want to spend up to $200 to rent the car, how many miles can you drive?

Instructional Task 2 (MTR.3.1)

The following function represents the value of a savings account at a local bank in Orlando, Florida, where t represents time in years, P represents the initial investment and r represents the investment return rate.

$$F(t) = P(1+r)^t$$

James' parents set up a savings account when he is 5 years old to help him pay for college tuition. They invest $1,000 at 5% interest.

Part A. How much money will be in the account when James is 19 years old?

Part B. How much money should his parents have invested if they wanted him to have $5,000 to start college?

Instructional Items

Instructional Item 1

Evaluate $f(6,7)$, when $f(x,y) = 8x + 3y$.

Instructional Item 2

Given $f(t) = \begin{cases} 16t, & if\ 0 \le t \le 1 \\ 6(t-1)+16, & if\ 1 < t \le 2 \\ 14(t-2)+22, & if\ 2 < t \le 3 \\ 8(t-3)+36, & if\ 3 < t \le 4 \end{cases}$, find $f(3)$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.F.3 *Create new functions from existing functions.*

MA.912.F.3.2

Benchmark

MA.912.F.3.2	Given a mathematical or real-world context, combine two or more functions, limited to linear, quadratic, exponential and polynomial, using arithmetic operations. When appropriate, include domain restrictions for the new function.

Benchmark Clarifications:

Clarification 1: Instruction includes representing domain restrictions with inequality notation, interval notation or set-builder notation.

Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.5.7
- MA.912.AR.9.10
- MA.912.F.1.2

Terms from the K-12 Glossary

- Domain
- Exponential function
- Linear function
- Polynomial
- Quadratic function

Vertical Alignment

Previous Benchmarks

- MA.7.AR.1
- MA.8.AR.1
- MA.912.AR.1.3, MA.912.AR.1.4, MA.912.AR.1.7

Next Benchmarks

Purpose and Instructional Strategies

In middle grades, students performed operations on linear expressions. In Algebra I, students performed addition, subtraction, multiplication and division with polynomials. In Math for Data and Financial Literacy, students combine two or more functions including linear, quadratic, exponential and polynomial, using arithmetic operations.

- When appropriate, the domain restrictions will be determined for the new function. Students will evaluate the solution when combining two functions for a provided input.
- In this benchmark, students will combine functions through addition, subtraction, multiplication, and division. This process will utilize their prior experience with polynomial arithmetic in MA.912.AR.1 from Algebra 1.
    - In mathematical contexts, combinations through addition may be represented as $(f+g)(x)$ or as $h(x) = f(x) + g(x)$.
    - In mathematical contexts, combinations through subtraction should be represented as $(f-g)(x)$ or as $h(x) = f(x) - g(x)$.
    - In mathematical contexts, combinations through multiplication should be represented as $(f \cdot g)(x)$ or as $h(x) = f(x) \cdot g(x)$.
    - In mathematical contexts, combinations through division should be represented as $\left(\frac{f}{g}\right)(x)$ or as $h(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$. Additionally for division, it can be represented using the division symbol.
- When combining functions by division, students will need to consider values in the domain of the quotient that should be restricted. Values that should be restricted are those that would cause the denominator to equal zero.
- Instruction includes representing domain, range and constraints using words, inequality notation, interval notation and set-builder notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of x, such that x is a real number."
    - Inequality notation
      If the domain is all values of x greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the range is all values of y less than or equal to zero, it can be represented as $\{y|y \leq 0\}$ and is read as "all values of y such that y is less than or equal to zero."
    - Interval notation
      If the domain is all values of x less than or equal to 3, it can be represented as $(-\infty, 3]$. If the domain is all values of x greater than 3, it can be represented as $(3, \infty)$. If the range is all values greater than or equal to -1 but less than 5, it can be represented as $[-1, 5)$.
- For this course, problems should focus on money and business.
    - For example, a business has to purchase merchandise to sell at their market. The business may use one function to represent the amount of merchandise they need

over the course of a year, and another function to represent the changing price of the merchandise over the course of a year.

- Students have previous knowledge of linear and quadratic functions, and should make the connection that by multiplying two linear functions, it results in a quadratic function.
    - For example, in order to determine revenue, one could multiply the function that represents price of a product by the function that represents quantity of a product that will be sold at that price.

Common Misconceptions or Errors

- Students may confuse the domain and range values in relationship to the input and output.
- Students may need additional practice on the operations with the functions.

Instructional Tasks

Instructional Task 1 (MTR.6.1)

A large business is evaluating their budget and trying to determine a function that represents their profits based on the price of one of their products.

Part A. If the company has a fixed expense of $235,000 per year and each product item costs $85 to produce, create a function for the company's annual expenses if they produce q items in a year.

Part B. The company has determined that the number of items, q, they can sell in a year at price p is $10000 - 23p = q$. Determine a domain for this function assuming that the number of items, q, is positive.

Part C. Using the information from Part B, create a function, $R(p)$, to represent the annual revenue, pq.

Part D. Using the information from Parts A and C, create a function that represents the annual profit based on the price, p.

Instructional Items

Instructional Item 1

Given $f(x) = -403x^2 + 12401x + 6302$ and $g(x) = 4.1x + 521$, find $(f - g)(x)$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.1 *Build mathematical foundations for financial literacy.*

MA.912.FL.1.1

Benchmark

MA.912.FL.1.1 Extend previous knowledge of operations of fractions, percentages and decimals to solve real-world problems involving money and business.

Benchmark Clarifications:
Clarification 1: Problems include discounts, markups, simple interest, tax, tips, fees, percent increase, percent decrease and percent error.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.2
- MA.912.FL.3
- MA.912.FL.4

Terms from the K-12 Glossary

- Percent of change
- Percent error
- Simple interest

Vertical Alignment

Previous Benchmarks

- MA.7.NSO.1.2
- MA.7.AR.3

Next Benchmarks

Purpose and Instructional Strategies

In middle grades, students worked with operations of fractions, percentages and decimals to solve real-world problems. In Math for Data and Financial Literacy, students utilize their understanding of operations and problem solving to solve real-world problems involving money and business. Throughout instruction, it will be important to help students connect the mathematical concepts to everyday experiences *(MTR.7.1)* as they validate conclusions by comparing them to a given situation.

- Instruction includes discounts, markups, simple interest, tax, tips, fees, percent increase, percent decrease and percent error *(MTR.7.1)*.
    - Markdown/discount is a percentage taken off of an original price. Instruction includes showing the connection between subtracting the calculated discount or taking the difference between 100% and the discount and multiplying that by the original price.
        - For example, if there was a 15% discount on an item that costs $15.99, students could take 85% of $15.99 or take 15% of $15.99 and subtract that value from the original price of $15.99.
    - Markup showcases adding a charge to the initial price. Markups are often shown in retail situations.
    - Simple interest refers to money you can earn by initially investing some money (the principal). The percentage of the principal (interest) is added to the principal making your initial investment grow. The simple interest formula ($I = prt$) calculates only the interest earned over time. Each year's interest is calculated from the initial principal, not the total value of the investment of that point in

time. The simple interest amount formula ($A = (1 + rt)$) calculates the total value of an investment over time. When using simple interest, provide the formula as students should not be expected to memorize this.

  - Tax, tips and fees are an additional charge added to the initial price. Students can add the calculated tax, tip or fee to the original price or add 1 to the tax, tip or fee and multiply it by the original price to reach the final cost.
    - For example, if there was a 6% sales tax on clothing and a t-shirt costs \$7.99. Students can add 100% to the 6% and multiply that value to \$7.99, or students can find 6% of the \$7.99 and add that to the original value of the t-shirt.
    - For example, if the bill is \$54.83, students can find 18% of the total and then add it back to the original.
  - Percent increase/decrease asks students to look for a percentage instead of a dollar amount. Percent error is a way to express the relative size of the error (or deviation) between two measurements.

$$\%\ error = \frac{|estimation - actual|}{actual} \times 100$$

Common Misconceptions or Errors

- Students may incorrectly truncate repeating decimals when problem solving.
- Students may incorrectly divide when the quotient is not a whole number.
  - For example, students may use the remainder of a problem as a decimal representation.
- Students may incorrectly place the decimal point when calculating with percentages. If students have discovered the shortcut of moving the decimal point twice, instruction includes understanding of how a percent relates to fractions and decimals.
- Students may forget to change the percent amount into decimal form (divide the percent by 100) when setting up an equation *(MTR.3.1)*.
- Students may incorrectly believe all percentages must be between 1 and 100%. To address this misconception, provide examples of percentages below 1% and over 100%.
- Students may incorrectly believe a percent containing a decimal is already in decimal form.
  - For example, emphasize that 43.5% is 43.5 out of 100 and dividing by 100 will provide the decimal form.
- In multiple discount problems, students may incorrectly combine the discounts instead of working them sequentially *(MTR.5.1)*.
  - For example, 25% off, then 10% off could incorrectly lead to 35% off rather than finding 25% off before calculating the additional 10% off.

Instructional Tasks

Instructional Task 1 (MTR.6.1)

Video Pro Shop and The Video Store both sold game systems for $450. In February, Video Pro Shop wanted to increase their profits so they increased the prices of their game systems by 15%. When this increase failed to bring in more money, they decreased their price by 10% in November. To beat their competitor who had increased prices, The Video Store decided to decrease their price of video games by 10% in March. However, when they started to lose money on the new pricing scheme, they increased the price of the game system in November by 15%.

Part A. If no other changes were made after November, which store now has the better price for the game system?

Part B. What is the difference between their prices?

Instructional Task 2 (MTR.7.1)

Sherri goes to dinner with 3 friends at a local restaurant in Floral City. The total bill was $82.45. The tax rate where the restaurant is located is 7.5% and they want to leave a 20% tip on the original total bill.

Part A. If they split the bill evenly, how much will each person pay, including tax and tip?

Part B. One of Sherri's friends has a $20 gift card and wants to use it to help with the dinner costs. If the gift card is applied to the entire bill before payment, how much will each person pay?

Instructional Items

Instructional Item 1

Joseph sells internet plans through phone call sales. He receives 11% commission on any sales up to $500. If he sells any plans over the $500 sales, he will earn 15% commission on those sales. If he sold $5,250 in one month, what was his commission for that month?

Instructional Item 2

A new clothing company has 30 employees, 40% of which are women. After 22 new employees joined the team; the percentage of women was increased to 50%. How many of the new employees are women?

Instructional Item 3

Part A. Mika takes out a loan that adds interest each year on the initial amount. What is the interest Mika will pay on the loan if he borrowed $15,000 at an annual interest rate of 4.5% for 15 years? (Use the formula $I = Prt$, where I is the interest, P is the principal or initial investment, r is the interest rate per year and t is the number of years.)

Part B. What will be the total amount Mika will have to pay back if the loan provider charges an additional $500 fee?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.1.2

Benchmark

MA.912.FL.1.2 Extend previous knowledge of ratios and proportional relationships to solve real-world problems involving money and business.

Example: A local grocery stores sells trail mix for $1.75 per pound. If the grocery store spends $0.82 on each pound of mix. How much will the store gain in gross profit if they sell 6.4 pounds in one day?

Example: If Juan makes $25.00 per hour and works 40 hours per week, what is his annual salary?

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.FL.2.4, MA.912.FL.2.6
- MA.912.FL.3
- MA.912.DP.1.2
- MA.912.DP.2.4
- MA.912.DP.3

Terms from the K-12 Glossary

- Proportional relationships

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3

Next Benchmarks

Purpose and Instructional Strategies

In grade 7, students have worked with ratios and proportional relationships to solve real-world problems. In Math for Data and Financial Literacy, students utilize their understanding of proportional relationships and problem solving to solve real-world problems involving money and business, including currency exchange, taxes and simple interest. Students also use concepts of proportionality when working with relative frequencies in data.

- Throughout instruction, it will be important to help students connect the mathematical concepts to everyday experiences *(MTR.7.1)* as they validate conclusions by comparing them to a given situation.
- Instruction for this benchmark includes opportunities to compare two different proportional relationships to each other. Allow various methods for solving, encouraging discussion and analysis of efficient and effective solutions *(MTR.4.1)*.
- Students should understand the difference between flat rate taxes and other tax rates. Flat rate taxes are proportional tax rates such as sales tax. Comparatively, a progressive tax is one that can vary such as an income-based tax rate increasing when income increases.

Common Misconceptions or Errors

- Students may not understand the difference between an additive comparison and a multiplicative comparison. To help address this misconception, instruction includes the understanding that proportions are multiplicative comparisons.
- Students may incorrectly set up proportions with one of the ratios having incorrect numbers in the numerator and denominator.
- Students may need information to clarify flat tax rates versus other tax rates. When

working with students related to business and taxes in this benchmark include discussions on flat tax rates.

Instructional Tasks

Instructional Task 1 (MTR.6.1)

An online clothing store, B's Boutique sells 24 pieces of clothing every 30 minutes, and an online athletic store sells 5 shirts every 12 minutes.

Part A. Estimate how long it will take each store to sell 100 pieces of clothing.

Part B. Once an online store sells 200 items, the parent company for the online site gives the selling company a bonus of 20% of their sales. Will either store make a 20% bonus in a 24-hour period?

Instructional Task 2 (MTR.5.1)

At a local farm in Ruskin, Florida, a box of tomatoes sells for $8.50.

Part A. How many boxes would they need to sell to reach the sales goal of $8,000?

Part B. If the farm currently employs 10 workers who can fill 285 boxes in 3 days, how many days will it take them to fill enough boxes for the farm to make more than $8,000 on the sales of the boxes of tomatoes?

Instructional Items

Instructional Item 1

At We Play Sports store, they try to keep their inventory at a ratio of 7 to 4 for used equipment to new equipment. If the total inventory amount for the new equipment is $50,250, what is the amount of inventory for the used equipment?

Instructional Item 2

The Easy Connect company makes straps to keep luggage closed. They make some with a snap lock and some with locks that are either a combination lock or work with a special luggage app. Based on sales, they typically make 20% with a snap lock, 55% with a combination lock and 25% with an app lock. If they typically sell 1,250 straps a month, how many would they expect to sell that have combination locking straps?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.1.3

Benchmark

MA.912.FL.1.3 Solve real-world problems involving weighted averages using spreadsheets and other technology.

Example: Kiko wants to buy a new refrigerator and decides on the following rating system: capacity 50%, water filter life 30% and capability with technology 20%. One refrigerator gets 8 (out of 10) for capacity, 6 for water filter life and 7 for capability with technology. Another refrigerator gets 9 for capacity, 4 for water filter life and 6 for capability with technology. Which refrigerator is best based on the rating system?

Connecting Benchmarks/Horizontal Alignment

Terms from the K-12 Glossary

- MA.912.FL.2.1, MA.912.FL.2.2, MA.912.FL.2.3
- MA.912.FL.4.3, MA.912.FL.4.4, MA.912.FL.4.5, MA.912.FL.4.6

Vertical Alignment

Previous Benchmarks
- MA.6.DP.1.2, MA.6.DP.1.6
- MA.7.DP.1.1, MA.912.DP.1.2
- MA.912.GR.3.1

Next Benchmarks

Purpose and Instructional Strategies

In middle grades, students have worked with the mean or average to solve real-world problems. In Geometry, students solved problems involving the weighted average of points on a line. In Math for Data and Financial Literacy, students use weighted averages in a variety of ways, including portfolios.

- To calculate the weighted average, one can begin by multiplying each value in the set by its assigned weight, then add up the products. Next, divide the products' sum by the total sum of all weights.
    - For example, a smoothie consists of 5 ounces of milk, 3 ounces of bananas, 2.5 ounces of strawberries and 0.75 ounces of whey protein powder. Milk costs $0.04 per ounce, strawberries costs $0.20 per ounce, bananas costs $0.03 per ounce and whey protein costs $0.80 per ounce. To determine the costs of the smoothie, students can find the weighted average cost per ounce of the ingredients, using the quantity of ounces as the weighted value for each ingredient.
        - $\left(\frac{0.04(5)+0.20(2.5)+0.03(3)+0.80(0.75)}{11.25}\right) = \frac{0.2+0.5+.09+0.6}{11.25} = 0.12$

        $0.12 represents the cost per ounce of the ingredients for the smoothie. If the smoothie is 12 ounces, then the cost of the ingredients is $1.44.
- Students should understand that when determining the weighted averages and the weights are percentages, then the percentages should add up to one. In these cases, there is no need to divide by the sum of the weights.
    - For example, a new employee has two evaluation performance ratings with their supervisor, one in April and one in December, with the April rating counting for 40% of their evaluation and the one in December counting for 60% of the evaluation. The April score is 78 and the December evaluation score is 96.
        - To determine the weighted average, students can multiply each score by its weight, then find the sum. Next, students can divide by the sum of the weights.

$$Weighted\ Average = \frac{78(0.40)+96(0.60)}{(0.40)+(0.60)}$$

$$= \frac{31.2+57.6}{1} = 88.8$$

- A weighted average can be used evaluate something (i.e., an item, person's performance, or investment earnings) whose value results from a combination of elements that have

different significance.
- Weighted averages can be computed using the SUM function or the SUMPRODUCT function in a spreadsheet.

Common Misconceptions or Errors

- Students may find the average versus the weighted average.
- Students may need support in understanding the varying weights for items in the data set.
- Students may incorrectly identify the cell ranges when using the spreadsheet to calculate the weighted average.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Star Coffee has 3 varieties of coffee that were sold at their store this year. The coffee that is locally grown in Florida is $6.50 a pound and they sold 100 pounds. The coffee from South America is $7.50 and they sold 75 pounds. The coffee from Jamaica is $12.50 and they sold 50 pounds. Calculate the weighted average to determine cost per pound. How does this value compare to the unweighted average?

Instructional Task 2 (MTR.4.1, MTR.7.1)

Ross just graduated from high school and decided to put his graduation money into four types of investments. He put:

- 40% of his money in online company investments with a return rate of 20%;
- 30% of his money in video gaming companies with a 5% return rate;
- 20% of his money in a startup company with a 5% return rate; and
- 10% of his money in a sports company with a 10% return rate.

Part A. Calculate the weighted average rate of return Ross would receive.
Part B. If Ross wanted to make a greater return, what suggestions would you give him?

Instructional Items

Instructional Item 1

An employee at an online company receives his bonus based on the following work requirements. The employee’s score is listed with the percentage weight in the table below. In order to receive the bonus, the employee must have an overall weighted average of 95%, will the employee receive the bonus?

Performance Requirement	Score (out of 100)	Weight
Online for 8 hours per day	90	60%
Attends training meetings	100	20%
Meets daily quotas	75	10%
Zero complaints received	100	10%

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.2 *Develop an understanding of basic accounting and economic principles.*

MA.912.FL.2.1

Benchmark

MA.912.FL.2.1 Given assets and liabilities, calculate net worth using spreadsheets and other technology.

Example: Jose is trying to prepare a balance sheet for the end of the year based on his profits and losses. Create a spreadsheet showing his liabilities and assets, and compute his net worth.

Benchmark Clarifications:
Clarification 1: Instruction includes net worth for a business and for an individual.
Clarification 2: Instruction includes understanding the difference between a capital asset and a liquid asset.
Clarification 3: Instruction includes displaying net worth over time in a table or graph.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1, MA.912.AR.1.2
- MA.912.FL.1.1, MA.912.FL.1.2
- MA.912.DP.1.2

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.7.NSO.2
- MA.912.DP.1.1

Next Benchmarks

Purpose and Instructional Strategies

In middle grades, students became fluent in operations with rational numbers and solved problems involving money. In Algebra 1, students worked with data in tables and graphs involving rational numbers. In Math for Data and Financial Literacy, students gain understanding of net worth for individual planning and for business.

- The following terms are used for the application of this benchmark:

Term	Definition	Example
Assets	• Items or resources controlled by an individual or a business • Must have an economic value • Value can be based on ability to generate cash flow in the future • Can be considered as capital assets, liquid assets, or something in between depending on its liquidity	Personal examples include cash, house, retirement account, car, etc. Business examples include machinery, buildings, accounts receivable, patents, etc.
Liability	• Something a person or business owes • Often referred to as a debt • Usually to be repaid in cash	Personal examples include mortgage, credit card debt, student loan, etc. Business examples include taxes owed, accounts payable, machinery loans, etc.

Capital Assets	• Assets that include property of any kind • Can be movable or immovable • Can be tangible or intangible • Can be fixed or circulating	Personal examples include property, car, jewelry, furniture, etc. Business examples include machinery, buildings, patents, etc.
Liquid Assets	• Assets that can be quickly converted to cash	Personal examples include savings and checking accounts, actual cash, bonds, etc. Business examples include stocks, accounts receivable, corporate bonds, etc.
Liquidity	• The ease or quickness with which the asset can be turned into cash	High liquidity examples include actual cash, money market accounts, cryptocurrency, etc. Low liquidity examples include real estate, valuable pieces of art or antiques, etc.

- Net Worth is determined by the difference of assets and liabilities.
- Instruction includes a variety of asset possibilities that could be included such as a value of home, car, computers, stocks, bonds, checking and savings accounts. Liabilities should include examples such as home mortgage, car payments, student loans and credit card debt.

Common Misconceptions or Errors

- Students may have misconceptions of what can be considered an asset beyond money or bank accounts, such as a valued collection or money in retirement accounts.
- The term liability should be discussed as it may be a new concept for students. Discuss items that can be a liability including debt that comes from other items beyond just living expenses.

Instructional Tasks

Instructional Task 1

Belinda is planning to move to Tampa for a job. She owns her car and has been working on paying off the townhouse and will need to make decisions on selling them to move. As she is planning, she wants to determine her net worth. Using the chart below of her major assets and liabilities, determine her net worth.

Assets	Liabilities
Checking Account Balance: $657 Savings Account Balance: $5,275 Money Market Account Balance: $6,200 Value of Laptop: $2,000 Value of Family China and Jewelry: $2,500 Value of Townhouse: $255,000 Value of Car: $10,500	Balance of Townhouse Mortgage: $51,500 Credit Cards (3 creditors): $3,345 Student Loan Balance: $41,000 Personal Loan: $3,500

Part A. Create a spreadsheet showing Belinda's liabilities and assets and compute Belinda's net worth.

Part B. What recommendations would you make to her if she wants buy a new condo in Tampa that will cost $175,000, without taking out a new mortgage, and maintain assets of at least $10,000.

Instructional Task 2

A start-up company that develops online games is working with their accountant to begin a budget for the next three years. Develop a list of items that the company could consider to be their capital assets and their liquid assets, and include potential values that would represent each item.

Instructional Items

Instructional Item 1

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.2.2

Benchmark

MA.912.FL.2.2 Solve real-world problems involving profits, costs and revenues using spreadsheets and other technology.

Example: A travel agency charges $2400 per person for a week-long trip to London if the group has 16 people or less. For groups larger then 16, the price per person is reduced by $100 for each additional person. Create an expression describing the revenue as a function of the number of people in the group. Determine the number of people that maximizes the revenue.

Benchmark Clarifications:
Clarification 1: Instruction includes the connection to data.
Clarification 2: Instruction includes displaying profits and costs over time in a table or graph and using the graph to predict profits.
Clarification 3: Problems include maximizing profits, maximizing revenues and minimizing costs.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.5
- MA.912.AR.3.8
- MA.912.AR.5.7
- MA.912.F.3.2
- MA.912.DP.2.4, MA.912.DP.2.8, MA.912.DP.2.9

Terms from the K-12 Glossary

- Data

Vertical Alignment

Previous Benchmarks

- MA.8.DP.1
- MA.912.AR.3.6, MA.912.AR.3.7
- MA.912.F.1.6

Next Benchmarks

Purpose and Instructional Strategies

In middle grades and Algebra I, students have worked with data using tables and graphs to solve real-world problems. In Math for Data and Financial Literacy, students utilize their understanding of data, tables, and graphs to predict profits and solve problems related to maximizing profits, maximizing revenues and minimizing costs.

- Throughout instruction, it will be important to help students connect the mathematical concepts to everyday experiences *(MTR.7.1)* as they validate conclusions by comparing them to a given situation.
- The following terms are used for the application of this benchmark:

Term	Definition related to business applications
Profit	A gain that represents the positive difference for a business when all expenses are subtracted from the total revenue. $P = R - E$ is the profit equation where P is representing the profit, R is the revenue and E represents expenses.
Costs	Cost represents the amount of money that is used by a business, such as the costs for production, services or development. Some

	costs can be variable, like labor and materials, or impacted by other factors such as inflation and wage rates.
Revenue	Revenue is the amount of income that a company generates by the selling of products or services. $R = pq$ is the revenue function with R representing the revenue, p representing the price of the product sold and the q representing the quantity of the products sold. Other revenue can come from non-sales income such as interest or large asset sales.

- During instruction, students will need to gain understanding of expenses and the types of expenses. Expenses can be variable or fixed. The Expense function is $E = V + F$ with E representing total expenses, V representing variable expenses and F representing fixed expenses.
  - For example, when a company makes a product it has fixed expenses, such as equipment or real estate, and variable expenses, such as labor and materials. These expenses can be variable because the number of items can be variable. So, the typical formula for expenses, E, can be found by the product of the number of items, q, and the cost per item, c, plus the fixed expenses, f, which can be represented by the equation $E = qc + f$.
- Using the understanding of expenses and revenue, instruction will include determining the difference between revenue and expenses to determine the profit or loss.
  - For example, if a company has a manufacturing cost per item of \$53 and sells that item for p dollars, and the company a has fixed cost of \$2750 per month, then the expression for the monthly profit, P, could be represented as $P = pq - 53q - 2750$ which is equivalent to $P = q(p - 53) - 2750$.
- Instruction includes understanding the relationship between the price per item, p, and the demand for that item, which is the number of items, q, that can be sold at that price. This relationship is called the demand function. In the simplest case, the demand function is a linear function.
  - For example, a company determines that the demand function for its product is $q = 1500 - 10p$. So, one can determine that the maximum demand (y-intercept) is 1500 because that is the demand when the price of the product is \$0. There will be no demand for the product (x-intercept) when the price is \$150.
  - For example, if the monthly profit of a product is represented by the expression $q(p - 53) - 2750$ and the demand function is $q = 1500 - 10p$, then the monthly profit, P, can be represented as a function of p: $P = (1500 - 10p)(p - 53) - 2750$. The company can maximize its monthly profit by choosing the price, p, corresponding to the vertex of this quadratic function.
- Students may need to verify the reliability of solutions and determine if the overall profit is correct based on revenue.

Common Misconceptions or Errors

- Students may need support in working with formulas or solving for a specific variable in a formula.
- Students may need review on graphing and relating the vertical and horizontal axes to real life problem solving.

Instructional Tasks

Instructional Task 1 (MTR.5.1)

The Newz U Can Use company designs new video reels to help older adults learn how to use their cell phones and other devices. Using a graphing calculator, graph the following profit equation for Newz U Can Use: $P = -300p^2 + 22{,}500p - 345{,}000$.

Part A. Determine the maximum profit for the profit equation.

Part B. What will be the price point at the maximum profit?

Instructional Task 2 (MTR.4.1, MTR.7.1)

Part A. Create a spreadsheet for a startup company that includes one month of income revenue from at least two sources, with a minimum of five costs/expenses.

Part B. Calculate the end balance to determine a profit or loss for the startup company at months end.

Part C. What information would you need to determine how to maximize the profit?

Instructional Task 3(MTR.4.1)

A company's revenue and expense functions are shown.

$$R = -250p^2 + 17{,}500p \qquad E = -850p + 150{,}000$$

Part A. Discuss with a partner why the expense, E, decreases as the price, p, increases. Create a scenario to justify your reasoning.

Part B. Determine the profit function for the company.

Part C. Determine a price that maximizes the profit.

Instructional Items

Instructional Item 1

Wrangler Ranch located in central Florida manufactures treats for livestock for sale at a local market. The expense equation for q bags of treats is $E = 5.25q + 20{,}000$. What is the cost per bag average for producing 500 bags to the nearest cent?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.2.3

Benchmark

MA.912.FL.2.3	Explain how consumer price index (CPI), gross domestic product (GDP), stock indices, unemployment rate and trade deficit are calculated. Interpret their value in terms of the context.

Benchmark Clarifications:

Clarification 1: Instruction includes the understanding that quantities are based on data and may include measurement error.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.1
- MA.912.DP.1.2

Terms from the K-12 Glossary

- Data

Vertical Alignment

Previous Benchmarks | **Next Benchmarks**

- MA.7.AR.3

Purpose and Instructional Strategies

In middle grades, students have worked with problems involving decimals and percentages. In Math for Data and Financial Literacy, students learn and explain how the consumer price index, gross domestic product, stock indices, unemployment rate and trade deficit are calculated. The interpretation of the values determined will be placed in the context of the situation.

- Instruction includes using students' understanding of decimals and percentages as they learn the definitions and interpret displays of various financial indices and rates.
- Instruction includes the understanding that financial indices and rates are often estimates and may have measurement error or be based upon incomplete data. When calculating and interpreting these, students may need to consider significant digits.
  - For example, when calculating the unemployment rate, the relevant quotient may produce a repeating decimal which would be rounded (or truncated) to reasonable number of significant digits (*MTR.6.1*).
- The Consumer Price Index (CPI) is also referred to as the cost-of-living index. The CPI serves as an indicator of the changes in the total cost of specific products of services based on inflation. The CPI is a measure that represents the change in the price of a market basket of consumer goods, which might include food, clothing, housing and other consumer expenses. The measurement of the CPI has limitations due to sampling error and price data errors.
  - To determine the CPI, the cost of products during a current period is divided by the cost of products in a prior time period (base year), then multiplied by 100.
  - For example, the table below shows the cost of a market basket of goods for calculation of the CPI. A typical household buys 2 loaves of bread, 3 gallons of milk and 4 boxes of cereal per week in the years 2017, 2018 and 2019.

Year	Price of Loaf of Bread	Price of Gallon of Milk	Price of Box of Cereal
2017	\$2	\$3	\$3
2018	\$3	\$3	\$4
2019	\$3	\$4	\$4

  To calculate the CPI for each of the years with 2018 as the base year, first calculate the cost of each weekly basket then calculate the CPI.

- For the purposes of this course, market baskets used for CPI calculations are based on a few items.
- The Gross Domestic Product (GDP) is the total market value of goods and services that are produced in a country during a specified timeframe. The GDP can be calculated in three ways: income, production and expenditures. Calculating the GDP using the expenditure method is the most commonly used method that includes adding the three users of goods and services: households, government and businesses.

$$GDP = Consumption + Investment + Government\ Spending + Net\ Exports$$

$$GDP = C + I + G + N_x$$

$C =$ private consumption expenditures by households

$I =$ investment by business

$G =$ government spending

N_x = the net exports (total exports minus total imports)

- Stock Indices are used to help investors determine current stock prices compared to past prices as the investors determine and calculate market performance. Stock Index is a benchmark to help gauge the movement of the market based on a batch of stocks. The most common method of calculation is to find the average of the stocks this can be direct or indirect to account for the weighting of the stocks.
    - For example, the Dow Jones Industrial Average is the index that includes 30 of the largest stocks in the market. The index calculation is the sum of the stock prices of the 30 companies divided by the divisor. The divisor can vary when the stock splits or if there are companies that are added or taken out of the index.
- Unemployment rate is a percentage of the total labor force that is unemployed, yet they are still actively looking for employment and are willing to work. To calculate the unemployment rate, divide the number of unemployed by the total number of people in the labor force then multiply by 100.
- Trade Deficit occurs when a country's imports exceed its exports. This also refers to when a country needs more that it can create, which in turn is a deficit. To calculate the trade deficit, subtract a country's exports from the total value of its imports.

Common Misconceptions or Errors

- Students may need support in calculations with percentages and changing large values from decimals to percentages.
- Students may need instruction on understanding the meaning of the values once calculated, and what the margin of error is as a factor in the impact on the solution.
- The vocabulary may be new to students and understanding of calculations and understanding the reliability of the results.
- Students may need support in following the CPI steps to solve related to the base year for calculation.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

The table below shows the cost of a market basket of goods for calculation of the CPI. In one household, they buy 3 rolls of paper towels, 2 gallons of milk and 4 boxes of cereal per week in years 2018, 2019 and 2020.

Year	Price of a Roll of Paper Towels	Price of Gallon of Milk	Price of Box of Cereal
2018	$1	$3	$3
2019	$1	$3	$4
2020	$2	$4	$4

Part A. Calculate the CPI in 2020 with 2018 as the base year.

Part B. Explain a possible reason for the increase from year 2019 to year 2020 that would explain the inflation in price of some products.

Instructional Items

Instructional Item 1

If the number of unemployed people for a country is 5.2 million and the total number in the labor force is 123.2 million, determine the unemployment rate as a percent.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.2.4

Benchmark

MA.912.FL.2.4 Given current exchange rates, convert between currencies. Solve real-world problems involving exchange rates.

<u>Benchmark Clarifications:</u>
Clarification 1: Instruction includes taking into account various fees, such as conversion fee, foreign transaction fee and dynamic concurrency conversion fee.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.4.4

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.6.AR.3.2, MA.6.AR.3.5
- MA.7.AR.3

Next Benchmarks

- MA.912.C.3.10

Purpose and Instructional Strategies

In grades 6 and 7, students worked with the concept of rate including, real-world contexts. Students determined a rate for a ratio of quantities with different units and calculated and interpreted the corresponding unit rate. Additional work involving ratios, rates and unit rates, including conversions within the same and different measurement system, was learned in grades 6 and 7. In Math for Data and Financial Literacy, students use this prior knowledge and understanding of rates to convert between currencies when given the current exchange rates and solve real-world problems involving exchange rates.

- While working with currency exchange rates, students should be aware of the various fees when doing a currency conversion including, foreign transaction fees and dynamic concurrency conversion fees.
- Currency is the system that is used in a country for money. The currency exchange rate is the rate used to convert from one country's currency to the currency of another country.
- Exchange Rates are the rates that are used to convert one currency to another. This can be an exchange from one country to another or another economic zone. Current rates are found on the internet, or by going to a bank prior to travel or when needing to exchange currency.
- The fees that may be incurred include the currency conversion fee from the payment processor or, if in an ATM network, the fee to convert one currency to another as a fee for the financial transaction of converting the currency. Another fee for currency conversion is the foreign transaction fee, which is a charge to you as a consumer by the issuer of your credit or debit card or the ATM network you may be using on the same transaction.

Common Misconceptions or Errors

- Students may use multiplication or division inappropriately in calculation when working with the exchange rates.
- Students may need help with the currency codes on the exchange rate charts to assist in

calculations.

- Students may not include the fees in the final solution for conversions.

Instructional Tasks

Instructional Task 1

An art class at Winstead High School has a group of seniors who plan to major in art and attend The Savannah College of Art and Design. Before attending college, the group is planning a trip to Europe to learn about the history of art and experience early artist work. In order to determine their trip costs, they need to determine the currency exchange costs.

Part A. If the trip to Europe will cost each student $2,350 in U.S. dollars, calculate the cost in for the trip Europe in Euros. Use the most recent currency conversion chart.

Part B. The currency and transaction fee combined is 3%. Determine the total fees.

Part C. How much will the students have to save in order to travel to Europe including the fees in U.S. dollars?

Instructional Items

Instructional Item 1

The DeMarco family lives in Buffalo, New York, and will be traveling to Niagara Falls on the Canada side for vacation. They filled up their car at a gas station in Buffalo for $3.35 per gallon. If the family fills up at gas station in Canada before heading home and the gas is 3.65 per liter, what would be the U.S. equivalent for the gas price in Canada?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.2.5

Benchmark

MA.912.FL.2.5 Develop budgets that fit within various incomes using spreadsheets and other technology.

Example: Develop a budget spreadsheet for your business that includes typical expenses such as rental space, transportation, utilities, inventory, payroll, and miscellaneous expenses. Add categories for savings toward your own financial goals, and determine the monthly income needed, before taxes, to meet the requirements of your budget.

Benchmark Clarifications:

Clarification 1: Instruction includes budgets for a business and for an individual.

Clarification 2: Instruction includes taking into account various cash management strategies, such as checking and savings accounts, and how inflation may affect these strategies.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.3.6, MA.912.FL.3.8
- MA.912.FL.4.4

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks | **Next Benchmarks**

- MA.7.NSO.2

Purpose and Instructional Strategies

In middle grades, students engaged in real-world problems involving money calculations. In Math for Data and Financial Literacy, students develop budgets with various income points, inflation and cash management strategies.

- Instruction includes the use of spreadsheets to develop the budget to complete the computations.
- Instruction includes budget terms to support student understanding such as income, savings, net income and expenses, including variable and fixed liabilities.
- Budgets developed for individuals can vary greatly depending on expenses at different stages of life. They can be developed by month as well as yearlong depending on the need of the individual. Instruction includes a monthly budget that can transition to a yearly budget to show costs that are semi-annual or one-time costs such as an insurance payment.
- A business budget is developed to track income and expenses to have a plan for spending, determining profit and growth. The budget will help the business with long range planning and predictions for the company. Business budgets have similar components to personal budgets such as revenue, fixed costs, variable costs, one-time expenses, cash flow and profit.

Common Misconceptions or Errors

- Students may need more information of what is included in a budget such as utility bills, cell phone and internet use, consumable items such as groceries, and larger items such as rent and car payments.
- Students can display budgets in multiple ways by using graphs such as a line graph or circle graph and may need instruction on what the graphs represent.
- Students may need instruction on understanding terms of bills within the budget, for example, monthly, quarterly, semi-annual or bimonthly payments.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Create a spreadsheet for an annual personal budget to include income, weekly and monthly expenses and other expenses. Consider the frequency of the expenses and items that you may have based on your living plans for the future.

Part A. Using a $2,500 bi-weekly income, determine total income first as you set up the spread sheet.

Part B. Include some weekly expenses such as food, eating out and entertainment.

Part C. Include monthly expenses such as mortgage or rent, utilities, car loans, insurance and savings.

Part D. Include any other expenses that could potentially be in your budget like additional insurance, vacation, repair savings for car or home, and medical.

Instructional Items

Instructional Item 1

ACE Painting Company is working on their monthly budget. They have 12 employees that paint for them and are hired as contractors each month depending on how many homes they paint per month. In their budget, will this be variable or fixed expense?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.2.6

Benchmark

MA.912.FL.2.6 Given a real-world scenario, complete and calculate federal income tax using spreadsheets and other technology.

Benchmark Clarifications:

Clarification 1: Instruction includes understanding the difference between standardized deductions and itemized deductions.

Clarification 2: Instruction includes the connection to piecewise linear functions with slopes relating to the marginal tax rates.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.9.10
- MA.912.FL.1.1
- MA.912.FL.3.7, MA.912.FL.3.8

Terms from the K-12 Glossary

- Linear function
- Piecewise function
- Slope

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.1

Next Benchmarks

Purpose and Instructional Strategies

In grade 7, students learned how use percentages and ratios to solve multi-step real world percent problems that included tax. In Math for Data and Financial Literacy, instruction extends this to calculate federal income tax, understand standard deductions, itemized deductions and marginal tax rates.

- Instruction includes the connection to piecewise functions as a way to represent tax brackets.
- Federal Income Tax is the tax that is collected by the Internal Revenue Service (IRS).
    - Tax Rate is the rate at which a business or a person is taxed as a percentage.
    - Marginal Tax Rates are taxes based on income brackets and is the tax rate paid on the next dollar of income. The marginal tax rate increases as the income increases and are separated into seven tax brackets in the United States.
    - Instruction should include an overview of tax documents including the basic forms for filing: W-2, 1099, 1040EZ, 1040, and Schedules A and B. Additionally, students should have opportunity for learning how to access and use the standard deduction chart and the itemized Schedule A for Form 1040.
    - Standard deduction is the deduction that is set by the government based on the filing status.
    - Itemized deductions are amounts that a person pays for state and local income, real estate or sales taxes, personal property taxes, mortgage interest, and some medical expenses. Itemized deductions also include contributions to charity.

Common Misconceptions or Errors

- Students may not have experience with income tax forms thus instruction should include the format understanding.
- Students may have misconceptions on how to calculate income and where to determine the income for tax purposes.
- Students may make errors in calculations when working with tax spreadsheets and forms.
- Students may make errors when working with the piecewise functions and the notation for writing them. This may include the correct decimal from the percentage being used. Additionally, students may need instruction related to determining the slope between two points and finding the value of b in $y = mx + b$ when working with piecewise functions.

Instructional Tasks

Instructional Task 1 (MTR.3.1)

Use the following information for Timothy and Lynn Crosby to compute their tax form 1040, including a Schedule A and Schedule B, and determine if they will have a refund or will owe taxes.

The Crosby's have 2 children and live in Florida with no state tax. Timothy works at a furniture design company with wages of $55,875 and his federal income tax withholdings were $8,755. Lynn works as a part-time paralegal. Her wages are $39,675 and her federal income tax withholdings are $7,458. Their bank interest received is $658 and they received $324 in stock dividends. Itemized deductions include:

- Medical/Dental $2,300
- Interest $4,250
- Contributions to charity $200
- Educational expense deductions $300

Instructional Task 2

Create a piecewise function for the tax, y, for a taxpayer who is filing single using the schedule chart.

Schedule X—Single

If line 3 is: Over—	But not over—	The tax is:	of the amount over—
$0	$9,950	**$0 + 10%**	**$0**
9,950	40,525	**995 + 12%**	**9,950**
40,525	86,375	**4,664 + 22%**	**40,525**
86,375	164,925	**14,751 + 24%**	**86,375**
164,925	209,425	**33,603 + 32%**	**164,925**
209,425	523,600	**47,843 + 35%**	**209,425**
523,600	and greater	**157,804.25 + 37%**	**523,600**

Instructional Items

Instructional Item 1

Sample Tax Schedule

Marginal Tax Rate	**Tax Brackets**	
	Over	*But Not Over*
10.0%	$0	$8,725
12.0%	$8,725	$32,500
22.0%	$32,500	$78,250
24.0%	$78.250	$125,700
35.0%	$125,700	$450,000

Using the sample tax schedule, what is the marginal tax rate for a taxpayer that has a taxable income of $52,000?

Instructional Item 2

Alexander completed his tax return and has calculated that his tax is $12,567. His employer withheld $10,458. Determine if Alexander will receive a refund or owe money back to IRS, and how much for each scenario?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3 *Describe the advantages and disadvantages of short-term and long-term purchases.*

MA.912.FL.3.1

Benchmark

MA.912.FL.3.1 Compare simple, compound and continuously compounded interest over time.

Benchmark Clarifications:
Clarification 1: Instruction includes taking into consideration the annual percentage rate (APR) when comparing simple and compound interest.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1
- MA.912.AR.2.5
- MA.912.AR.5.7
- MA.912.AR.10.1, MA.912.AR.10.2

Terms from the K-12 Glossary

- Exponent
- Exponential function
- Linear function

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students worked with linear and exponential functions. In Math for Data and Financial Literacy, students utilize these function types to explore three types of interest. They will also explore an introduction to limits as they discover the number e.

- Throughout instruction, it will be important to help students recognize patterns and structure (*MTR.5.1*) as they explore scenarios to lead them to generate the formulas for each type of interest.
- Instruction includes student exploration and construction of each interest formula.
    - For simple interest, lead students to generate $I = prt$ by exploring patterns. Consider the following:
        - Jalen invests \$100 in a bank account that pays 5% simple interest each year. How much interest does Jalen earn in 15 years? What is the total value of his investment in 15 years?
        - Have students begin exploration by creating a table like the one below.

Year	Total Interest Earned	Total Amount
1	$100(0.05)$	$100(0.05) + 100$
2	$100(0.05) \times 2$	$100(0.05) \times 2 + 100$
3	$100(0.05) \times 3$	$100(0.05) \times 3 + 100$
4	$100(0.05) \times 4$	$100(0.05) \times 4 + 100$
t	$p(r) \times t$ or prt	$p(r) \times t + p$ or $prt + p$ or $p(1 + rt)$

        - Be sure to highlight the difference between the simple interest formula and those for compound interest. The formula $I = prt$ only calculates the interest, whereas the compound interest formulas calculate the worth of the total investment. Students should remember to use the formula $A = p(1 + rt)$ when calculating total investments in simple interest scenarios.

- For compound interest, lead students to generate $A = P\left(1 + \frac{r}{n}\right)^{nt}$ by exploring patterns. Consider the following:
  - Jalen invests $100 in a bank account that pays 5% interest compounded quarterly. How much interest does Jalen earn in 15 years? What is the total value of his investment in 15 years?
  - Guide students through an exploration beginning with the simple interest formula.
    - The interest for one quarter would be

$$I = prt$$
$$I = (100)(0.05)\left(\frac{1}{4}\right)$$
$$I = 100\left(\frac{0.05}{4}\right)$$

    - The total amount of the investment after one quarter would then be

$$A_1 = 100 + 100\left(\frac{0.05}{4}\right)$$
$$A_1 = 100\left(1 + \frac{0.05}{4}\right)$$

    - The value of the investment after 2 quarters, A_2, would be

$$A_2 = A_1 + A_1\left(\frac{0.05}{4}\right)$$
$$A_2 = A_1\left(1 + \frac{0.05}{4}\right)$$
$$A_2 = 100\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)$$
$$A_2 = 100\left(1 + \frac{0.05}{4}\right)^2$$

    - The value of the investment after 3 quarters, A_3, would be

$$A_3 = A_2 + A_2\left(\frac{0.05}{4}\right)$$
$$A_3 = A_2\left(1 + \frac{0.05}{4}\right)$$
$$A_3 = 100\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)$$
$$A_3 = 100\left(1 + \frac{0.05}{4}\right)^3$$

    - The value of the investment after 4 quarters, A_4, would be

$$A_4 = A_3 + A_3\left(\frac{0.05}{4}\right)$$
$$A_4 = A_3\left(1 + \frac{0.05}{4}\right)$$
$$A_4 = 100\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)$$

$$A_4 = 100\left(1+\frac{0.05}{4}\right)^4$$

- At this point, student should see the connection between the exponent 4 and the number of quarters the investment has accumulated interest. Speed the exploration up by moving to 8, 12 and 16 quarters to build the idea of the exponent being the product of the number of years and the number of times the investment compounds each year.
  - After 8 quarters (2 years), A_8, would be

$$A_8 = 100\left(1+\frac{0.05}{4}\right)^8$$

$$A_8 = 100\left(1+\frac{0.05}{4}\right)^{4(2)}$$

  - After 12 quarters (3 years), A_{12}, would be

$$A_{12} = 100\left(1+\frac{0.05}{4}\right)^{12}$$

$$A_{12} = 100\left(1+\frac{0.05}{4}\right)^{4(3)}$$

- Students should now see the pattern emerge for the compound interest formula. Have them replace the numbers in the equation to generate $A = P\left(1+\frac{r}{n}\right)^{nt}$.
- Be sure to highlight the difference between the simple interest formula and those for compound interest. The formula $I = prt$ only calculates the interest, whereas the compound interest formulas calculate the worth of the total investment. Students should remember to use the formula $I = p(1+rt)$ when calculating total investments in simple interest scenarios.

- For continuously compounded interest, lead students to understand $A = pe^{rt}$ by introducing a short exploration of limits. Consider the following:
  - Jalen invests $100 in a bank account that pays 5% interest, compounded continuously. How much interest does Jalen earn in 15 years? What is the total value of his investment in 15 years?
    - Exploration of the number e can be tough for students. Continuously compounding interest introduces the idea of limits, which may be a first experience for some students.
    - Have students informally discuss *(MTR.4.1)* the meaning of simple limits such as $\lim_{x\to\infty}\frac{1}{x}$, $\lim_{x\to\infty}2^x$, $\lim_{x\to\infty}1^x$ and $\lim_{x\to\infty}\frac{9x-7}{3x+12}$
    - Guide students to understand why the continuous compound interest formula contains a special letter that represents the limit involved when n is continuous.
    - Have students calculate the limit below and label the result as e.
      - $\lim_{x\to\infty}(1+\frac{1}{x})^x \approx 2.718281828$ which is equivalent to e.

        - Help students see the similarity in the limit structure and the compound interest formula.
    - Introduce the continuous compound interest formula as $A = Pe^{rt}$.
- Once students understand each interest formula, guide them to use those formulas in spreadsheet programs to explore various investment comparisons over time.

Value of Total Investment Over Time						
	Years Invested	**Simple Interest**	**Compound Interest (quarterly)**	**Compound Interest (monthly)**	**Compound Interest (daily)**	**Compound Interest (Continuous)**
Principal $3,000 Interest 4%	1	$3,120.00	$3,121.81	$3,122.22	$3,122.43	$3,122.43
	2	$3,240.00	$3,248.57	$3,249.43	$3,249.85	$3,249.86
	3	$3,360.00	$3,380.48	$3,381.82	$3,382.47	$3,382.49
	4	$3,480.00	$3,517.74	$3,519.60	$3,520.50	$3,520.53
	5	$3,600.00	$3,660.57	$3,662.99	$3,664.17	$3,664.21
	6	$3,720.00	$3,809.20	$3,812.23	$3,813.70	$3,813.75
	7	$3,840.00	$3,963.87	$3,967.54	$3,969.33	$3,969.39
	8	$3,960.00	$4,124.82	$4,129.19	$4,131.31	$4,131.38
	9	$4,080.00	$4,292.31	$4,297.41	$4,299.90	$4,299.99
	10	$4,200.00	$4,466.59	$4,472.50	$4,475.38	$4,475.47
	11	$4,320.00	$4,647.95	$4,654.71	$4,658.01	$4,658.12
	12	$4,440.00	$4,836.68	$4,844.35	$4,848.10	$4,848.22
	13	$4,560.00	$5,033.07	$5,041.72	$5,045.94	$5,046.08
	14	$4,680.00	$5,237.43	$5,247.13	$5,251.86	$5,252.02
	15	$4,800.00	$5,450.09	$5,460.90	$5,466.18	$5,466.36
	16	$4,920.00	$5,671.39	$5,683.39	$5,689.24	$5,689.44
	17	$5,040.00	$5,901.67	$5,914.94	$5,921.41	$5,921.63
	18	$5,160.00	$6,141.30	$6,155.92	$6,163.06	$6,163.30
	19	$5,280.00	$6,390.66	$6,406.73	$6,414.56	$6,414.83
	20	$5,400.00	$6,650.15	$6,667.75	$6,676.33	$6,676.62

- While using spreadsheet technology, as students learn to reference other cells in their formulas, there will be cases where they want a single cell's value (i.e., the principle of an investment) to remain in the formula as they copy it to multiple rows of a table. To achieve this, students should type a "$" symbol in front of the part of the cell name they want to remain static.
    - For example, when copying formulas that reference cell $A1$ across multiple rows/columns, using $\$A1$ will allow the rows to change, using $A\$1$ will allow the columns to change, and using $\$A\$1$ will maintain that specific cell in every row/column.
- Many institutions advertise investments by using the annual percentage yield (APY) rather than the annual percentage rate (APR) since it is greater for investments that compound multiple times annually. Students should be careful when analyzing investments to see which percentage is being advertised.
- When using spreadsheet technology for analysis, multiple problems can develop as students create and enter formulas for calculations. Write sample formula entries for compound interest, $= 2000 * (1 + 0.04/12)\text{^}(12 * 5)$, on the board for them to emulate to help prevent this. Note that many programs require a $*$ symbol to denote multiplication. Placing two variable (or cells) side by side may generate a REF error.

Common Misconceptions or Errors

- For simple interest scenarios, students may forget to add the principal to the interest they calculate using the simple interest formula when calculating the total amount of an investment over time.
- For compound interest scenarios, students may forget to subtract the principal from the total amount they calculate using a compound interest formula when calculating the total interest earned over time.
- Given the complexity of the formulas used in this benchmark, be sure to calculate one example by hand and by technology to confirm any formulas entered into spreadsheet technology are correct.

Instructional Tasks

Instructional Task 1 (MTR.2.1, MTR.4.1)

Four students make $500 investments for a period of 20 years.

- Jasmine earns simple interest at a 13% APR.
- Brayden’s investment earns 6.8% APR, compounded monthly.
- Dionne’s investment earns 7% APR, compounded semi-annually.
- Arthur’s investment earns 6.5% APR, compounded continuously.

Part A. Predict which student’s investment will have the greatest value in 20 years.
Part B. Predict which student’s investment will have the least value in 20 years.
Part C. Use a spreadsheet program and graphing technology to explore the value of these investments annually from year 1 to year 20.
Part D. Which investment would be best at year 16?
Part E. Choose which investment you would want. Explain your reasoning.

Instructional Items

Instructional Item 1

To the nearest cent, how much more does an investment of $2,500 earn over 12 years, compounded continuously at 2.5%, than a $2,500 investment over 12 years, compounded quarterly at 2.5%?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.2

Benchmark

MA.912.FL.3.2 Solve real-world problems involving simple, compound and continuously compounded interest.

Example: Find the amount of money on deposit at the end of 5 years if you started with $500 and it was compounded quarterly at 6% interest per year.

Example: Joe won $25,000 on a lottery scratch-off ticket. How many years will it take at 6% interest compounded yearly for his money to double?

Benchmark Clarifications:

Clarification 1: Within the Algebra I course, interest is limited to simple and compound.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1, MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.5.7
- MA.912.AR.10.1, MA.912.AR.10.2

Terms from the K-12 Glossary

- Exponent
- Exponential function
- Linear function

Vertical Alignment

Previous Benchmarks

- MA.912.FL.3.4

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students explored and compared investments involving simple and compound interest, and explained the relationships between interest and linear and exponential growth. In Math for Data and Financial Literacy, students use their knowledge of these interest formulas to solve real-world problems.

- Investments include single deposit investments as well as periodic investments.
  - Single deposit investments occur when the investor deposits a single sum of money into an investment. No further deposits are made by the investor. The formulas students engaged in MA.912.FL.3.1 represent single deposit investments.
  - Periodic investments occur when the investor deposits funds into the investment over regular intervals. This changes the formula used to calculate the worth of the investment at a given time to:

$$A = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt} - 1\right)}{\frac{r}{n}}$$

- Guide students to create a table like the one below to explore the impacts of periodic investments.
  - Samantha explores the impact of three different interest rates on a periodic investment of $500 per year over 20 years.

Years	Total Principal Invested	Total Amount Compounding Annually at 2.3%	Total Amount Compounding Annually at 5.7%	Total Amount Compounding Annually at 11.9%

1	$500.00	$500.00	$500.00	$500.00
2	$1,000.00	$1,011.50	$1,028.50	$1,059.50
3	$1,500.00	$1,534.76	$1,587.12	$1,685.58
4	$2,000.00	$2,070.06	$2,177.59	$2,386.16
5	$2,500.00	$2,617.68	$2,801.71	$3,170.12
6	$3,000.00	$3,177.88	$3,461.41	$4,047.36
7	$3,500.00	$3,750.97	$4,158.71	$5,029.00
8	$4,000.00	$4,337.25	$4,895.76	$6,127.45
9	$4,500.00	$4,937.00	$5,674.82	$7,356.62
10	$5,000.00	$5,550.55	$6,498.28	$8,732.05
11	$5,500.00	$6,178.22	$7,368.68	$10,271.17
12	$6,000.00	$6,820.32	$8,288.70	$11,993.44
13	$6,500.00	$7,477.18	$9,261.15	$13,920.65
14	$7,000.00	$8,149.16	$10,289.04	$16,077.21
15	$7,500.00	$8,836.59	$11,375.51	$18,490.40
16	$8,000.00	$9,539.83	$12,523.92	$21,190.76
17	$8,500.00	$10,259.25	$13,737.78	$24,212.46
18	$9,000.00	$10,995.21	$15,020.84	$27,593.74
19	$9,500.00	$11,748.10	$16,377.02	$31,377.40
20	$10,000.00	$12,518.30	$17,810.51	$35,611.31

- Problem types may require students to calculate the total value of an investment at given points in time, or could require them to calculate the principle, interest rate or the time needed to achieve a particular investment goal.
- Many institutions advertise investments by using the annual percentage yield (APY) rather than the annual percentage rate (APR) since it is greater for investments that compound multiple times annually. Students should be careful when analyzing investments to see which percentage is being advertised.
- Given the complexity of the formulas used in this benchmark, be sure to calculate one example by hand and by technology to confirm any formulas entered into spreadsheet technology are calculating correctly.
- When using spreadsheet technology for analysis, multiple problems can develop as students create and enter formulas for calculations. Write sample formula entries for compound interest, $= 2000 * (1 + 0.04/12)\hat{}(12 * 5)$, on the board for them to emulate to help prevent this. Note that many programs require a $*$ symbol to denote multiplication. Placing two variable (or cells) side by side may generate a REF error.
- While using spreadsheet technology, as students learn to reference other cells in their formulas, there will be cases where they want a single cell's value (i.e., the principle of an investment) to remain in the formula as they copy it to multiple rows of a table. To achieve this, students should type a "$" symbol in front of the part of the cell name they want to remain static.
  - For example, when copying formulas that reference cell $A1$ across multiple rows/columns, using $\$A1$ will allow the rows to change, using $A\$1$ will allow the columns to change, and using $\$A\$1$ will maintain that specific cell in every row/column.

Common Misconceptions or Errors

- The formula for periodic investments is complex. Students should solve portions of the formula over several steps to ensure accurate calculations. Recording these steps in writing can help identify errors for miscalculations.

Instructional Tasks

Instructional Task 1

Three students all aspire to be millionaires by retirement. They each set up periodic investments for 40 years each.

- Amie considers investing $700 per month into in investment that earns 3.4% interest, compounded annually.
- Trey considers investing $500 per month into in investment that earns 6.3% interest, compounded annually.
- Keisha considers investing $250 per month into in investment that earns 9.1% interest, compounded annually.

Part A. Why do you think each student is receiving different interest rates?
Part B. Which students would become millionaires?
Part C. For students who would not earn a million dollars by the end of the 40-year investment, what monthly deposit would they need to make in their investment to achieve that goal?

Instructional Items

Instructional Item 1

Sheri wants to invest $5,000 in an account that pays 2.3% interest, compounded quarterly. How many years would it take for her investment to double?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.3

Benchmark

MA.912.FL.3.3 Solve real-world problems involving present value and future value of money.

Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1, MA.912.NSO.1.6, MA.912.NSO.1.7
- MA.912.AR.2.5
- MA.912.AR.5.7
- MA.912.AR.10.1, MA.912.AR.10.2

Terms from the K-12 Glossary

- Exponent
- Exponential function
- Linear function

Vertical Alignment

Previous Benchmarks

Next Benchmarks

- MA.912.FL.3.4

Purpose and Instructional Strategies

In Algebra I, students explored and compared investments involving simple and compound interest and solved real world problems related to those types of investments. In Math for Data and Financial Literacy, students use their new knowledge of these interest formulas to solve real-world problems involving the present value or future value of investments.

- The present value of investments is the sum of money that must be invested to achieve a specific future goal.
  - To calculate the present value of an investment, students should solve their interest formula for the variable P.
  - For single deposit investments, this leads to:
$$A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$
  - For periodic investments, this leads to:
$$A = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}} \rightarrow P = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt}-1}$$
- The future value of investments is the dollar amount that will accumulate over a given time-period when that sum is invested. Most of students' work with interest formulas in MA.912.FL.3.1 and MA.912.FL.3.2 has focused on exploring future values of investments.
  - For single deposit investments students use $A = P\left(1+\frac{r}{n}\right)^{nt}$.
  - For periodic investments students use $A = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}}$.
- Given the complexity of the formulas used in this benchmark, be sure to calculate one example by hand and by technology to confirm any formulas entered into spreadsheet technology are calculating correctly.

- When using spreadsheet technology for analysis, multiple problems can develop as students create and enter formulas for calculations. Write sample formula entries for compound interest, $= 2000 * (1 + 0.04/12)\string^(12 * 5)$, on the board for them to emulate to help prevent this. Note that many programs require a $*$ symbol to denote multiplication. Placing two variable (or cells) side by side may generate a REF error.
- While using spreadsheet technology, as students learn to reference other cells in their formulas, there will be cases where they want a single cell's value, like the principle of an investment, to remain in the formula as they copy it to multiple rows of a table. To achieve this, students should type a "$" symbol in front of the part of the cell name they want to remain static.
    - For example, when copying formulas that reference cell $A1$ across multiple rows/columns, using $\$A1$ will allow the rows to change, using $A\$1$ will allow the columns to change, and using $\$A\$1$ will maintain that specific cell in every row/column.

Common Misconceptions or Errors

- The formula for periodic investments is complex. Students should solve portions of the formula over several steps to ensure accurate calculations. Recording these steps in writing can help identify errors for miscalculations.

Instructional Tasks

Instructional Task 1

Idris wants to save $20,000 over the next 6 years to buy her first car. She researches investment options and finds three investments to consider. Investment A offers 1.3% interest compounded monthly. Investment B offers 1.9% interest compounded semiannually. Investment C offers 2.1% interest compounded annually. For each investment, how much would Idris need to deposit each year to meet her goal? Which investment is the better deal?

Instructional Items

Instructional Item 1

Pasha invests $6,000 each year into a mutual fund that averages 11.6% interest, compounded annually. Assuming the average interest rate doesn't change, how many years will it take for her reach $1,000,000? How many years will it take for her to reach $2,000,000?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.5

Benchmark

MA.912.FL.3.5 Compare the advantages and disadvantages of using cash versus personal financing options.

Example: Compare paying for a tank of gasoline in the following ways: cash; credit card and paying over 2 months; credit card and paying balance in full each month.

Benchmark Clarifications:
Clarification 1: Instruction includes advantages and disadvantages for a business and for an individual.
Clarification 2: Personal financing options include debit cards, credit cards, installment plans and loans.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.3.9
- MA.912.FL.4.3

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students are introduced to the advantages and disadvantages of various financing options for individuals as well as businesses.

- Instruction includes comparisons between cash purchases and purchases using debit cards, credit cards, installment plans or loans. The table below has some example comparisons of advantages and disadvantages for purchasing options.

Purchase Option	Advantages	Disadvantages
Cash	• Avoid debt • Statistically spend less on purchases (cash is hard to save and, therefore, hard to spend) • No interest or penalties	• Purchases can only occur when the full amount of cash needed is saved • Limited to in-person transactions; doesn't work online • Money cannot be used to track purchases • Large purchases require a person to hold on to a large amount of cash
Debit Cards	• Avoid debt • Protected against unauthorized purchases • Can be used in place of a credit card for online purchases • Provide a record of purchases • Convenient/quick access to funds for purchases	• Purchases can only occur when the buyer has a sufficient balance in their checking account • Possible to overdraw the account, leading to penalties
Credit Cards	• Purchase products/services without having the full amount of the purchase on hand • Protected against unauthorized	• Interest and penalties for users who do not pay off their balance in a timely manner • Statistically spend more on

	purchases • Can be used for online purchases • Provide a record of purchases • Reward programs such as cash-back or airline miles	purchases (transactions are less visible than spending cash, easier to spend) • Immediate purchase power leads to more "impulse buys"
Installment or Deferred Payment Plans	• Purchase products/services without having the full amount of the purchase on hand • Break down the purchase into smaller payments over an extended period compounded monthly	• With added interest, more is paid for the product/service than would be by using cash/debit card • Penalties for users who do not pay off their balance in a timely manner
Loans	• Purchase products/services without having the full amount of the purchase on hand • Break down the purchase into smaller payments over an extended period • Enable the purchase of large value products (i.e., homes)	• With added interest, more is paid for the product/service than would be by using cash/debit card • Penalties for users who do not pay off their balance in a timely manner

- Most credit cards charge interest to the average daily balance of the account over the billing period. Students analyzing the cost of purchasing larger items with a credit card will need to factor other items that may be purchased with the credit card into account when analyzing repayment calculations.

Common Misconceptions or Errors

- Depending on the terms of a loan or credit card, students may need to convert the APR into a monthly or daily interest rate. Be sure to have students double check the terms of the finance option before beginning calculations.

Instructional Tasks

Instructional Task 1 (MTR.4.1)

Hugo wants to purchase new carpet for multiple rooms in his house. The cost of the carpet and installation is estimated to be $8,200. Hugo considers three purchasing options:

1) purchasing the carpet with cash from his savings account (current savings balance: $10,000);
2) purchasing the carpet with a credit card that has 26.99% APR for purchases and paying it off over 12 months; and
3) using an installment plan that requires a 20% down payment and monthly payments of $389.77 for 18 months.

Discuss the advantages and disadvantages of each option.

Instructional Items

Instructional Item 1

Deshawn is considering buying a new TV for his home. Describe one disadvantage of using an installment plan for the purchase rather than using cash.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

Benchmark

MA.912.FL.3.6 Calculate the finance charges and total amount due on a bill using various forms of credit using estimation, spreadsheets and other technology.

Example: Calculate the finance charge each month and the total amount paid for 5 months if you charged $500 on your credit card but you can only afford to pay $100 each month. Your credit card has an annual finance rate of 17.99%.

Benchmark Clarifications:

Clarification 1: Instruction includes how annual percentage rate (APR) and periodic rate are calculated per month and the connection between the two percentages.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.1

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.912.FL.3.4

Next Benchmarks

Purpose and Instructional Strategies

In Algebra I, students compare simple and compound interest. In Math for Data and Financial Literacy, students calculate finance charges and total amounts due on a bill for these forms of credit.

- Instruction of this benchmark includes how APR and periodic rate are calculated per month and the connection between the two percentages. Students should also have an understanding of common financial terms.
- For loans and installment plans, the key to finding the finance charge is calculating the total paid over the term of the loan or plan and then subtracting the amount of the purchase. To find the total amount paid, students should use the formula below to calculate the monthly payment for the loan or installment plan and then multiply this amount by the number of months of the loan or installment plan.

Monthly Payment (Installment) Formula

$$M_1 = \frac{P(r)(1+r)^{12t}}{(1+r)^{12t}-1}$$

$$M_2 = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t}-1}$$

$P =$Principal amount of the loan

$R =$Annual Percentage Rate

$r = \frac{R}{12}$, periodic monthly interest rate

$M =$Monthly payments

$t =$Number of years of loan

  - For example, Sasha is buying her first home and plans to take out a 30-year, $235,000 mortgage with an APR of 4.2% to make the purchase. What is the total amount she will spend for the home and how much is the finance charge?

- One of the first steps would be to calculate the monthly payment for her mortgage.

$$M = \frac{\$235{,}000(.0035)(1+.0035)^{12(30)}}{(1+.0035)^{12(30)}-1} \approx \$1{,}149.19$$

- To determine the total amount paid on the loan, students will multiply the monthly payment by the total number of payments $12 \cdot 30 = 360$

$$\$1{,}149.19 \times 360 = \$413{,}708.50$$

- Finally, to determine the finance charge (total **cost** of loan) subtract the purchase price from the total amount paid on the loan.

$$\$413{,}708.50 - \$235{,}000 = \$178{,}708.50$$

- Finance charges on a credit card are calculated at the end of each billing cycle (usually monthly). The APR for the credit card is applied to the average daily balance of the credit card over the billing cycle, not the final balance at the end of the month. Finding the average daily balance of the credit card will require knowing the balance of the card for each day of the month.
  - For example, for a 30-day billing cycle, the daily balances for a credit card that has an APR of 24.99% might be:

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
$502	$502	$502	$502	$502	$502	$561	$561	$561	$643
Day 11	Day 12	Day 13	Day 14	Day 15	Day 16	Day 17	Day 18	Day 19	Day 20
$643	$643	$643	$643	$643	$643	$643	$643	$701	$701
Day 21	Day 22	Day 23	Day 24	Day 25	Day 26	Day 27	Day 28	Day 29	Day 30
$701	$701	$701	$701	$701	$728	$728	$728	$728	$728

  - The sum of the balances is $19,029. Divide this by 30 to get $634.30, the average daily balance.
  - To calculate the finance charge for the billing cycle, multiply the average daily balance by the periodic interest rate. Periodic rates are equal to the annual percentage rate divided by the number of periods interest is applied during the year. In this case, the APR should be divided by 12 since the periodic rate is applied monthly.
    - $r = 24.99\% \div 12 = 2.0825\%$
    - $\$634.30 \times 0.020825 \approx \$13.21$
  - So, the finance charge for this billing cycle would be $13.21.
- Since the finance charges of credit cards are based on the average of the balances across the billing cycle, the timing of a purchase in the billing cycle directly affects the finance charge at the end of the month. Large purchases made at the beginning of a billing cycle would increase the average daily balance more than those made toward the end of a billing cycle. This variability makes interpreting word problems involving credit cards difficult. For the purposes of this course, assume credit card purchases in provided examples are made on the first day of the billing cycle, with an initial balance of $0 on the card, and are the only purchases for the billing period.
- Most credit cards charge interest to the average daily balance of the account over the billing period. Students analyzing the cost of purchasing larger items with a credit card will need to factor other items that may have been or may be purchased with the credit

card into account when analyzing repayment calculations.

Common Misconceptions or Errors

- Given the complexity of the formulas used in this benchmark, be sure to calculate one example by hand and by technology to confirm any formulas entered into spreadsheet technology are calculating correctly.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Calculate the finance charge each month and the total amount paid for 10 months if you charged $2,500 on your credit card that has an annual finance rate of 21.99%.

Part A. If you can only afford to pay $250 each month, how long would it take to pay off the charge and fees accrued?

Part B. Is it possible to pay off the charge and fees accrued within the 10 months?

Instructional Task 2 (MTR.4.1)

Omar takes out a 30-year mortgage for $250,000 at a rate of 4.62%. Shanti takes out a 15-year mortgage for $250,000 at a rate of 4.68%.

Part A. Calculate the total finance charges of each loan.

Part B. Compare the two mortgages.

Instructional Items

Instructional Item 1

Jackie is going to purchase a used car for $16,499 with a 5-year loan that has an APR of 3.99%. What is the finance charge for the loan if she makes monthly payments?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.7

Benchmark

MA.912.FL.3.7 Compare the advantages and disadvantages of different types of student loans by manipulating a variety of variables and calculating the total cost using spreadsheets and other technology.

Benchmark Clarifications:
Clarification 1: Instruction includes students researching the latest information on different student loan options.
Clarification 2: Instruction includes comparing subsidized (Stafford), unsubsidized, direct unsubsidized and PLUS loans.
Clarification 3: Instruction includes considering different repayment plans, including deferred payments and forbearance.
Clarification 4: Instruction includes how interest on student loans may affect one's income taxes.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.10.2
- MA.912.FL.1.1
- MA.912.FL.2.6

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students are introduced to different types of student loans and analyze the advantages and disadvantages of each type. They learn about various repayment plans of these loans and their impact they can have on tax returns.

- Instruction includes allowing students to research the current information in regards to student loans as changes can be made on a yearly basis.
- Students should understand the difference between direct and indirect student loans, and the different types of each.
    - Direct loans are issued and managed by the U.S. Department of Education and are backed by the federal government. Federal direct loans are fixed-rate and can be used to pay for the costs of education, including tuition, room and board, books, and other education-related expenses. Indirect loans are obtained from a third-party lender.
        - **Direct Subsidized (Stafford) Loans**
          Direct subsidized loans are only available to undergraduates that demonstrate a financial need. They typically have lower interest rates that other student loans. They are called subsidized loans because the government pays the loan interest while a student is in school at least half-time and continues to pay it for six-months after the student leaves school. The government will also pay the loan interest during a period of deferment.
        - **Direct Unsubsidized (Stafford) Loans**
          Direct Unsubsidized Loans are available to both undergraduate and graduate students. The financial need of the student is not a factor in

obtaining the loan. They feature higher interest rates than direct subsidized loans and the government does not pay any interest on the loan. Repayment for these loans does not begin until 6 months after graduation, leaving school or dropping below halftime enrollment. Interest for these loans begins to accrue immediately upon receipt. When payments for the loan begin, any interest that accrues is *capitalized*, meaning it is added to the principal of the loan. Future interest will be calculated from this new principal for the remainder of the loan.

        - **Direct PLUS Loans**
          PLUS stands for Parent Loan for Undergraduate Students. Direct PLUS Loans are used by parents of a prospective student to pay for college expenses. The terms of the loan depend on the parent's credit standing.
        - **Indirect Unsubsidized Loans**
          Indirect Unsubsidized Loans are granted by financial institutions other than the federal government. These loans have less restrictions that direct loans, but typically come with higher interest rates.

- Students should understand that with any direct loan, there are options for repayment.
    - **Standard Repayment Plan**
      Most students are automatically enrolled in this plan by default. The repayment term is typically 10 years, with equal payments each month.
    - **Graduated Repayment Plan**
      This plan begins with lower payments that gradually increase across a 10-year term. Ultimately, students pay more than they would with the Standard option because of how payments are structured.
    - **Income-Based Repayment Plan**
      Sets payments based off a percentage of students' monthly discretionary income. Payment terms can stretch up to 25 years. This repayment plan can lower monthly payments, but the longer terms mean more interest will be paid overall compare with a standard repayment plan. This plan is not available for PLUS loans.
    - **Extended Repayment Plan**
      This plan is available to borrowers with more than $30,000 in direct loans. It allows students to pay off loans over 25 years by making fixed or graduated payments.
- Instruction includes student understanding of various terms related to student loans, including deferment period and forbearance.
    - **Deferment Period**
      A deferment period is a period during which a borrower does not have to make payments on a loan. Lenders may grant deferment during times of financial hardship as an alternative to default. Interest may continue to accrue during a deferment period, which means the interest is added to the amount due at the end of the deferment period.
    - **Forbearance**
      Forbearance is the temporary postponement/reduction of loan payments. The terms of a forbearance agreement are negotiated between the borrower and the lender. The borrower must demonstrate their need for postponing payments, such as financial difficulties brought on by a major illness or the loss of a job as well as

their trustworthiness to be able to resume payments once the financial difficulty is over. Lenders are often willing to negotiate forbearance agreements to try and prevent the losses they incur when a borrower defaults on a loan. Once the forbearance period is over, borrowers begin the process of repaying the missed/deferred payments.

- Instruction allows students to discuss the difference between deferment periods and grace periods (*MTR.4.1*). A grace period is a length of time after a due date that a borrower can make a payment without incurring a penalty. Grace periods are typically set in days whereas deferment periods are usually expressed in months
- Students should discuss the various tax implications that can be associated with different student loans options (MA.912.FL.2.6). Most direct student loan interest is tax-deductible since you're repaying the government, they don't tax you on the interest you repay them.

Common Misconceptions or Errors

- Students may think that interest does not accrue for unsubsidized loans during the grace period.
- Students may forget to capitalize grace period interest when calculating monthly payments for unsubsidized loans.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Mitchell is planning to attend a 4-year university to earn a bachelor's degree and then enroll in a second university for two additional years to earn his master's degree. For both degrees, he takes out direct unsubsidized loans. The first loan is for $57,000 at 3.82% over 10 years, and the second loan is for $31,000 at 5.28% over 10 years. Both loans have a grace period until 6 months after graduation.

Part A. If Mitchell decides to pay interest-only payments on his loan during his grace periods, how much will he pay each month for each loan?

Part B. If Mitchell does not make any interest payments during his grace periods, what will be the new principals of each loan when the grace period expires?

Part C. How much money would Mitchell save on each loan if he makes interest payments while attending school?

Instructional Task 2 (MTR.4.1, MTR.7.1)

Naomi has finished a 4-year degree using a direct subsidized student loan of $27,000 at 3.9%. Nearing the end of her grace period, Naomi considers two repayment plans. The standard repayment plan features payments of $272 per month over 10 years. A graduated payment plan over 10 years starts with payments of $152 per month and increases the payment 31.5% every two years.

Part A. Calculate the total amount repaid under each plan. Which plan repays the least amount?

Part B. Under which conditions would the graduated payment plan be the better option? Share your opinion with a partner.

Instructional Items

Instructional Item 1

Nina has just graduated from high school and is making plans to attend a 4-year university in the fall. She has been approved for a $27,000 direct subsidized student loan for 10 years at 3.73% interest.

Part A. How much will the U.S. Department of Education subsidize in interest costs during her 4.5-year nonpayment period?

Part B. What is the total amount will Nina pay over the course of the 10-year loan?

Instructional Item 2

Juan has finished a 4-year degree using a direct unsubsidized student loan of $32,000 at 3.77%. Nearing the end of his grace period, Juan applies for and receives a 36-month deferment with the option to pay monthly interest payments or have the interest capitalized at the end of the deferment period. How much money would Juan save over the course of the entire loan by paying interest payments during his deferment period?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.8

Benchmark

MA.912.FL.3.8 Calculate using spreadsheets and other technology the total cost of purchasing consumer durables over time given different monthly payments, down payments, financing options and fees.

Example: You want to buy a sofa that cost $899. Company A will let you pay $100 down and then pay the remaining balance over 3 years at 15.99% interest. Company B will not require a down payment and will defer payments for one year. However, you will accrue interest at a rate of 18.99% interest during that first year. Starting the second year you will have to pay the new amount for 2 years at a rate of 26 % interest. Which deal is better and why? Calculate the total amount paid for both deals.

Example: An electronics company advertises that if you buy a TV over $450, you will not have to pay interest for one year. If you bought a 65’ TV, regularly $699.99 and on sale for 10% off, on January 1st and only paid $300 of the balance within the year, how much interest would you have to pay for the remaining balance on the TV? Assume the interest rate is 23.99%. What did the TV really cost you?

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.2.5
- MA.912.FL.3.6

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students analyze multiple payment options for a purchase.

- Instruction allows for students to explore multiple real-world scenarios involving consumer durables (items that are durable and last long periods of time before needing to be replaced). Purchases to explore include but are not limited to furniture, appliances, electronics, tools, computers, or automobile purchases. When considering these purchases, allow students to research for real-time financing options.
    - For example, if you're considering the purchase of a couch, search the financing options of three major furniture providers and let students calculate which has the better deal.
- Given the volume of consumer durable purchases students may make in the future, have students consider the benefits of saving up for purchases as compared to financing them. Many of these purchases are not immediate needs and can be delayed until students can pay for them in full.
- In addition to comparing purchasing options for a given consumer durable, have students factor potential monthly payments into a hypothetical household budget (MA.912.FL.2.5). Students should consider whether having other financed items (student loans, mortgages, car loans) would they have the margin to make a given purchase.
- While exploring financing options, look for fine print that says that minimum monthly payments may be less than the amount needed to pay off the loan by the end of the interest free period. Ask students why lenders would set the payments up this way.

Common Misconceptions or Errors

- Students may miss that many finance options back charge interest for the full beginning principle, even if some of the principle has been paid off during an "interest free" period.
- Students may not use periodic interest in their calculations and may use the APR.

Instructional Tasks

Instructional Task 1 (MTR.4.1)

Sasha wants to purchase a new bedframe and mattress for her bedroom that costs $3975. The furniture store offers an 18-month financing plan where no interest is charged if the purchase is paid off in 18 months. If the purchase is not fully repaid, interest will be charged on the full purchase price at an APR of 29.99%.

Part A. If Sasha makes $100 payments each month, what will the balance of the loan be after month 18?

Part B. What would her minimum monthly payment need to be to not incur any interest charges?

Part C. How much would she have saved in interest by making this payment?

Part D. What would be the advantages and disadvantages of using this finance option versus saving up and paying with cash?

Instructional Task 2 (MTR.5.1)

Audrey is shopping for a new car. The model she wants is $28,000. Explore the following financing options and discuss the advantages and disadvantages of each.

Part A. Audrey's local credit union offers a 48-month or a 60-month loan at 2.99%. Calculate the estimated monthly payment and total interest charge for each option.

Part B. The dealership offers a 72-month loan at 3.74%. Calculate the estimated monthly payment and total interest charge Audrey would pay.

Part C. The dealership offers an 84-month loan at 4.82%. Calculate the estimated monthly payment and total interest charge Audrey would pay.

Part D. If the car loses 20% of its value in the first year of ownership and around 15% each year afterwards, calculate the amount of time for each purchase option it would take before the worth of the car exceeded the balance of the car loan.

Instructional Items

Instructional Item 1

Alfonso considers buying a new gaming desktop computer at a local electronics store for $1449.99. He calculates he has $100 per month in discretionary funds he can use to pay for the computer. He considers two purchase options:

1) using his credit card (with an APR of 18.99%) to make the purchase or
2) using a 12-month interest free loan from the electronics store with an APR of 27.99% applied to original purchase price if not fully paid in 12 months.

Which option would be better? Explain your reasoning.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.9

Benchmark

MA.912.FL.3.9 Compare the advantages and disadvantages of different types of mortgage loans by manipulating a variety of variables and calculating fees and total cost using spreadsheets and other technology.

Benchmark Clarifications:

Clarification 1: Instruction includes understanding various considerations that qualify a buyer for a loan, such as Debt-to-Income ratio.

Clarification 2: Fees include discount prices, origination fee, maximum brokerage fee on a net or gross loan, documentary stamps and prorated expenses.

Clarification 3: Instruction includes a cost comparison between a higher interest rate and fewer mortgage points versus a lower interest rate and more mortgage points.

Clarification 4: Instruction includes a cost comparison between the length of the mortgage loan, such as 30-year versus 15-year.

Clarification 5: Instruction includes adjustable rate loans, tax implications and equity for mortgages.

Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.2
- MA.912.FL.1.1
- MA.912.FL.2.5

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students explore multiple factors that relate to mortgages and examine key factors that influence the ability to obtain mortgages in general.

- Point out for students that mortgages are typically the largest loans most adults seek to obtain. Given the scale of the loan, banks are careful to research the capacity and trustworthiness of buyers to be able to repay the loan. If buyers default on mortgage payments, the bank could lose large amounts of money. Given this risk, lenders use various considerations that qualify buyers for a loan.
    - **Debt-to-Income Ratio**
      This ratio examines a buyer's margin in their monthly budget to be able to pay for a mortgage. Two ratios are commonly used. Front-end ratios examine the total monthly housing costs for a buyer, including the current mortgage/rent payment, mortgage insurance, homeowner's insurance, and property taxes.
      $$\frac{(total\ monthly\ housing\ costs)}{(monthly\ gross\ income)} = \text{Front-end Ratio}$$
      Lenders typically look for front-end ratios that are less than 28 percent to approve a mortgage application. Back-end ratios explore the other recurring debts a buyer has each month outside of their housing expenses. These include installment loans, car payments and credit card payments.
      $$\frac{(total\ monthly\ recurring\ debts)}{(monthly\ gross\ income)} = \text{Back-end Ratio}$$
      Lenders typically look for back-end ratios that are less than 36 percent to approve a mortgage application.
    - **Credit Score**
      Lenders lean heavily on credit scores to determine the trustworthiness of buyers to repay the mortgage. Buyers with poor credit history will have higher interest rates for loans due to the greater risk incurred by the lender.
    - **Down Payment**
      A down payment is an initial payment made at the time of purchase for the home before entering into a mortgage. Down payments increase the trustworthiness of the buyer to the lender because buyer has invested their money in the purchase as well as the lender's money, which makes it more likely for the buyer to repay the loan. Down payments can lower interest rates for the buyer and increase the chance of being approved for a mortgage. Down payments can be any percentage of the purchase price, but down payments of 20% or more allow buyers to avoid mortgage insurance.
    - **Work History**
      Lenders will check buyers work history to ensure a steady track record of income over a sustained period of time. This helps ensure their ability to repay the loan.
    - **Home Quality**
      Lenders will also require a home inspection before providing a mortgage. If a buyer defaults on the loan, the lender will eventually sell the home to regain as much of the loan amount as possible. For that reason, lenders want to make sure homes are in good condition and will achieve high selling prices should the need arise.
- When considering purchasing a home, calculating monthly payments based off of the purchase price is not enough. Additional fees can come into play when purchasing a

home that need to be considered. These fees are commonly known as *closing costs*.

- **Discount Prices**
  Discount prices depend on the use of discount points (also known as mortgage points). Typically, the cost of one mortgage point equals 1% of the loan amount (some lenders allow for fractions of a percent), and this single point lowers the interest rate of the mortgage by about 0.25%. For example, if the mortgage amount is $250,000 at a 3.5% mortgage rate, a buyer might purchase one mortgage point for $2,500 (paid at closing) to get a 3.25% interest rate instead. The $2,500 paid at closing would be considered the "discount price."
- **Origination Fee**
  This fee is paid to the lender to cover the administrative costs for processing the mortgage application. These fees are typically 0.5% to 1% of the loan amount.
- **Maximum Brokerage Fee**
  Mortgage brokers can be used to help home buyers find and secure a mortgage loan. They explore multiple lenders and loan options in pursuit of the best deal for the buyer. Their service typically comes at a cost of 1% to 2% of the loan amount. Depending on the quality of the loan they generate, this might be a worthwhile investment.
- **Documentary Stamps**
  Homes come with a title, which identifies the owner or the home and property. When purchasing a home, the home title will be transferred to a new owner. This documentation requires the payment of a transfer fee. In Florida, this fee is called the "Florida documentary stamp tax." This fee is calculated at $0.35 for every $100 of the purchase price.
- **Prorated Expenses**
  At closing, there are several expenses that the seller of the home has been paying that need to be transferred to the buyer. Proration is the process of dividing various property expenses between the buyer and seller in a way that allows each party to only pay for the time they own the property. These expenses include property taxes, homeowner's insurance, home owner's association dues and mortgage interest. As an example, sellers who have paid for a full year's worth of property tax but are selling their home before the end of the year will have the "unused" amount added to closing costs so they get it refunded to them. They buyer pays this amount at closing and, in essence, pays the property tax for the remainder for the year.

- After introducing the idea of mortgage points to students, have them perform a cost comparison between a higher interest rate and fewer mortgage points versus a lower interest rate and more mortgage points. Highlight the idea that the use of mortgage points, especially on longer term loans, can generate significant savings in interest payments.
  - For example, Tonya takes out a $260,000 30-year fixed-rate mortgage at 4.2%. Her lender offers an interest rate of 3.7% if she purchases 2 mortgage points. On a $260,000 loan, two points would cost an additional $5,200 at closing. Is this a good deal?
  - If Tonya chooses not to buy mortgage points, her interest rate will remain at 4.2%. Over 30 years, without paying down the loan early, the cost of the loan, with interest, is $455,739.98. However, if she purchases two mortgage points, she

would pay $429,333.18 over the life of the loan. This means that buying mortgage points saved Tonya $26,406.80 over the course of her mortgage.

- Instruction includes a cost comparison between lengths of mortgage loans, such as 30-year, 20-year and 15-year. Students should discover that shorter term loans will have higher monthly payments but smaller total interest charges. Have students discuss *(MTR.5)* the advantages and disadvantages of each. Consider guiding students to examine following comparison:
    - Compare the monthly payment and total interest payment for a $320,000 mortgage at 3.41% for 15-, 20- and 30-year terms.

	15-Year	**20-Year**	**30-Year**
Monthly Mortgage Payment	$2,273.51	$1,841.11	$1,420.91
Total Interest Paid	$89,231.31	$121,865.47	$191,529.35

- Instruction addresses adjustable rate loans, tax implications and equity for mortgages.
    - **Adjustable Rate Loans**
      Adjustable rate mortgages (ARMs) carry interest rates that shift (adjust) at a pre-arranged frequency over time. They typically have terms of 30 years. These mortgages can start with lower interest rates than fixed rate mortgages, but over time will adjust to have higher interest rates than fixed rate mortgages.
    - ARMs have additional variables that require the introduction of some new terminology:
        - **Adjustment Frequency**
          The time between interest rate adjustments. Typically, this will start with a long introductory interval such as 5 or 10 years and then shift to shorter intervals such as 6 months. For this reason, frequencies are communicated with two numbers such as 7y/6m, which represents a 7-year introductory interval followed by 6-month adjustment intervals.
        - **Adjustment Index**
          This is a benchmark used to set the interest rates. Many lending institutions use the Secured Overnight Financing Rate (SOFR) as their index.
        - **Margin**
          Buyers who use an ARM agree to pay an interest rate that is a constant percentage above the Adjustment Index. This amount is called the margin.
        - **Caps**
          The limit an interest rate can increase each adjustment period. Some loans feature a higher initial adjustment cap followed by a smaller cap for the shorter adjustment intervals. Other loans may show an overall cap for the life of the loan. Add this to the introductory interest rate to find the ceiling.
        - **Ceiling**
          The highest an interest rate allowed for the life of the loan.
    - Adjustable Rate Mortgages are rarely a good idea for long-term home buyers since the interest rates will eventually rise above fixed rates. ARMs are typically purchased by home buyers who plan to sell their home before the end of the introductory interval.
    - **Tax Implications**

Buying and owning a home does bring some tax benefits for the buyer/owner. Mortgage point costs, mortgage interest payments, and property taxes are tax deductible and can reduce the total amount of income you pay taxes on each year.

  - **Equity**
    Equity refers to the difference in what a buyer owes for their home and the home's present value. For a buyer who pays a 20% down payment for a home worth $200,000, they would have $40,000 in equity in the home at the time of purchase. Their equity in the home increases as mortgage payments reduce the principal of the mortgage. Equity can also increase as the value of the home increases. If home values fall, equity in the home decreases as a result.

- Instructions includes other mortgages such as Federal Housing Administration (FHA) and U.S. Department of Agriculture (USDA) and situations when one may need to have one of these loans.

Common Misconceptions or Errors

- Students may not consider homeowner's insurance, property taxes or Private Mortgage Insurance (if making a down payment less that 20%) when considering how much of a mortgage they can afford for a given budget.
- Students may not consider that both front-end and back-end ratios are considered by lenders. Budgets that have too large of a back-end ratio will not be eligible for many loans even if the front-end ratio is under 28%.
- If considering an ARM, student may assume the adjustment index rate will remain constant over time. Heavy research should be done on the historical fluctuation of adjustment index rates before assuming the risk of an ARM.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.6.1)

You're planning to buy your first home. The house you want is listed at $240,000. You've saved enough to make a 10% down payment. Closing costs will total 4% of the purchase price and PMI will cost you $117 each month. Assuming your budget will allow for up to $1,500 a month in mortgage expenses after paying property taxes and homeowner's insurance, analyze the following purchase options and list the advantages and disadvantages of each.

- A 30-year fixed rate mortgage at 2.75%.
- A 30-year fixed rate mortgage at 2.25% after using 2 mortgage points.
- A 20-year fixed rate mortgage at 2.625%.
- A 7y/6m ARM (30-year term) with a 2.125% introductory rate, a margin of 2.75% over SOFR (currently at 0.05%), 5% first adjustment cap, 1% subsequent adjustment cap and a lifetime cap of 5%.

Instructional Items

Instructional Item 1

You and your spouse are applying for a mortgage with a local bank, who will check your front-end and back-end ratio before approving the loan. Your combined take home income is $98,756. The monthly mortgage for the house you want is $1,496 each month. The annual property taxes would be $1,825 and your homeowner's insurance premium would cost $1,114 each year. You have two car payments, one $578 per month and the other $456 per month, and one student loan payment of $272 each month. Based on these expenses, would the bank approve your loan? Why or why not?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.10

Benchmark

MA.912.FL.3.10	Analyze credit scores qualitatively. Explain how short-term and long-term purchases, including deferred payments, may increase or decrease credit scores. Explain how credit scores influence buying power.

Benchmark Clarifications:

Clarification 1: Instruction includes how each of the following categories affects a credit score: past payment history, amount of debt, public records information, length of credit history and the number of recent credit inquiries.

Clarification 2: Instruction includes how credit score affects qualification and interest rate for a home mortgage.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.1.1

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students explore how credit scores are calculated and multiple factors that cause them to increase or decrease. They also learn about the impact credit scores have over buying power.

- Instruction begins with a discussion *(MTR.4.1)* among students regarding what they think lenders look for when deciding if a potential borrower is trustworthy. Organize their thoughts on your whiteboard or a piece of chart paper to reflect on later in the lesson. After the discussion, inform student that credit scores are the metric lenders commonly use to gauge the trustworthiness of a borrower.
- Instruction includes exploring how credit scores impact buying power so students can identify the benefits of maintaining a high credit score. Have students explore various online loan calculators that feature a credit score field they can change. They should quickly notice that interest rates for loans increase as credit scores decrease. As a class, calculate the difference in total interest payments for a given loan depending on the credit score of the borrower. Use the information you find online or consider the following example.
    - Two families are applying for 30-year loans to purchase homes that each will cost

$240,000 after down payments. The families' income and monthly expenses are nearly identical. The major difference is their credit history. The Johnson's have a credit score of 631 and receive an offer at 4.312%. The Thompson's have a credit score of 803 and receive an offer at 3.216%. If they both take the out their respective loans, how much more in interest will the Johnson's pay due to poor credit?

- Once students gain an appreciation for how credit scores affect their future finances, begin instruction regarding how they are calculated.
  - Credit scores are calculated from your credit report. Your credit report lists what types of credit you use (credit cards, installment loans, mortgage loans, etc.), the balances on these accounts, the length of time your accounts have been open, and whether you've paid your bills on time. It tells lenders how much credit you've used and whether you're seeking new sources of credit. There are three credit bureaus that generate credit reports: Equifax, TransUnion and Experian.
  - The most common credit score is calculated by Fair, Isaac and Company (FICO) and is used by a majority of lenders. Five key categories of information from your credit report are used to generate a FICO score.
    - **Payment History**
      This category counts for around 35% of your overall score as lenders are very concerned with the likelihood borrowers will repay their loans. It shows how consistently you've paid your accounts over your credit history. A history of on-time payments will cause this category to increase and a history of late payments or delinquent accounts will cause it to decrease. Deferring payments does not have an impact on this category, since the lender is giving written permission to defer.
    - **Amount of Current Debt**
      This category counts for around 30% of your overall score as consumers who take out too much credit are more likely to make late or missed payments. Keeping high balances on credit cards will cause this category to increase and keeping low credit card balances reducing balances of other loans will cause it to decrease. Deferring payments can affect this category since account balances are not decreasing over a period of time.
    - **Length of Credit History**
      This category counts for around 15% of your overall score. Borrowers with longer credit histories, especially positive histories, receive higher scores.
    - **Types of Credit Held (stated as Public Records Information in benchmark clarification)**
      This category counts for around 10% of your overall score. Higher scores for this category go to borrowers who successfully manage different types of loans. Opening too many types of loans too quickly can negatively impact the final category.
    - **Number of Recent Credit Inquiries or New Credit**
      This category counts for around 10% of your overall score. Opening too many new credit accounts too quickly or having too many new credit inquiries represents greater risk to lenders and will lower scores in this

category. One exception to this is rate shopping for low mortgage, auto, or student loans. The credit inquiries needed to explore rates for these are often treated as a single inquiry if they are done within a 30-day window.

- Once students understand how credit scores are calculated, explore how short-term and long-term purchases may increase or decrease credit scores.
  - Short-term purchases using revolving accounts, such as credit cards, or short installment loans can help build a positive payment history and length of credit history which can increase credit scores if your balances are low. Opening too many accounts to quickly or carrying high balances can lower your credit score.
  - Long-term purchases, such as home, student or auto loans, can build a positive payment history and length of credit history which can increase credit scores if your balances are consistently decreasing. Deferred payments on these loans can keep your balances high and lower your scores. Taking on too many larger, long-term purchases will lower your credit score.
    - Since home loans are typically the largest loans taken by most consumers, they rely heavily on credit scores to set interest rates and other loan parameters. Most conventional mortgage lenders require at least a 620 credit score to obtain a loan.

Common Misconceptions or Errors

- Students may assume that each category of a credit score calculation works independently of the others and seek to positively impact one category without considering the impact on other categories.
  - For example, opening a new line of credit could increase the Types of Credit category but it also lowers your average credit history across your accounts and increases the amount of total debt. Students should consider the impact of decisions across all 5 categories when making financial decisions.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Katy currently rents an apartment. She recently applied for a home loan but was not eligible for a low interest rate due to a low credit score. Rather than take out a mortgage at a high interest rate, she decides to delay a home purchase until she can raise her credit score. Below is a list of her current credit accounts.

- Credit Card A - $5,000 limit, $2,143 current balance, 3 late payments in the last 12 months
- Credit Card B - $3,000 limit, $1,687 current balance, 2 late payments in the past 12 months
- Installment Loan A (Furniture) - $1,809 current balance
- Car Loan A - $15,756 current balance
- Student Loan A - $23,875 current balance

What steps could Katy take over the coming months to raise her credit score and become eligible for lower interest rates?

Instructional Items

Instructional Item 1

Generally, which of the following most influences your credit score?

a. Having a long credit history
b. Maintaining low balances on credit accounts
c. Keeping a positive payment history
d. Opening several new credit accounts

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.3.11

Benchmark

MA.912.FL.3.11 Given a real-world scenario, establish a plan to pay off debt.

Example: Suppose you currently have a balance of $4500 on a credit card that charges 18% annual interest. What monthly payment would you have to make in order to pay off the card in 3 years, assuming you do not make any more charges to the card?

Benchmark Clarifications:
Clarification 1: Instruction includes the comparison of different plans to pay off the debt.
Clarification 2: Instruction includes pay off plans for a business and for an individual.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.1.1
- MA.912.FL.2.5

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students explore different strategies to pay off debt.

- Instruction begins with planning to pay off a single debt then moving to paying off multiple debts within a personal budget plan.
- Calculating the monthly payment needed to pay off the credit card in a specified amount of time requires a new formula. (Note: there are several websites that provide calculators that use this formula as well.) In the formula below, B is the existing credit card balance, M is the monthly payment, r is the interest rate expressed as a decimal, n is the number of times the interest is compounded annually and t is the length of time in years.

$$M = \frac{B\left(\frac{r}{n}\right)}{\left(1 - \left(1 + \frac{r}{n}\right)^{-nt}\right)}$$

- After students have calculated monthly payments needed to pay off debt in a desired timeframe, they will need to devise a plan in their monthly budget to allocate the necessary amount for debt reduction. Provide sample budgets like the one below and have students discuss *(MTR.4.1)* what changes could be made to make room for new debt, reducing debt or other adjustments.

Monthly Budget					
Charity		**Food**		**Personal**	
Charity and Offerings	$200.00	Groceries	$500.00	Childcare	$400.00
		Restaurants	$150.00	Toiletries	$50.00
Saving				Hair Care	$50.00
Emergency Fund	$250.00	**Clothing**		Subscriptions	$35.00
Retirement	$500.00	New Clothes	$50.00	Gifts	$50.00
College	$50.00	Cleaning/Laundry	$25.00	Spending Money	$80.00
				Pet Supplies	$50.00
Housing		**Transportation**		Music/Technology	$15.00
Mortgage/Rent	$1,400.00	Gas	$350.00	Miscellaneous	$75.00
Real Estate Taxes	$200.00	Car Repair	$50.00	Other	
Repairs/Maint.	$50.00				
		Insurance		**Debt**	
Utilities		Life	$25.00	Car Payment 1	$400.00
Electricity	$196.00	Homeowner/Renter	$79.00	Car Payment 2	$250.00
Water	$21.00	Auto	$231.00	Credit Card 1	$50.00
Trash	$28.00	Identity Theft	$13.00	Student Loan 1	$150.00
Phone/Mobile	$175.00			Other	$ -
Internet	$55.00	**Recreation**			
Cable	$40.00	Entertainment	$40.00		
Alarm	$40.00	Vacation	$ -		
Medical/Health					
Medications	$62.00				
Doctor Bills	$100.00				
Health Visits	$40.00				
Monthly Income	$6,575.00				
Projected Expenses	$6,575.00				

As students discuss, guide them to recognize a few key points like some categories in the budget are easier to adjust (i.e., entertainment or restaurants) than others (i.e., mortgage or electricity).

- Instruction includes prompting questions that allow students to discuss various situations that may arise within trying to establish pay off plans.
  - Even when establishing a payoff plan, why might it be good to maintain an emergency fund?
  - What additional expenses might occur during your payoff plan?
  - What are some ways you might increase your income temporarily to help pay off debt faster?

- How could your budget differ if your only debt was your mortgage?
- Instruction includes making the connection to long-term retirement savings when deciding to reduce retirement funds for a payoff plan. Guide students to explore the Periodic Investment formula using a spreadsheet and a cell reference for the monthly contribution (which allows students to change it easily) to allow students to quickly see the exponential effects of reducing their monthly retirement savings.
- Instruction includes exploring various time frames when determining a payoff plan.
  - For example, monthly payments of $412.56 may be required to pay off a card in a year. Have students determine if adjustments to the budget be made to achieve this.

Common Misconceptions or Errors

- Given the complexity of the formulas used in this benchmark, be sure to calculate one example by hand and by technology to confirm any formulas entered into spreadsheet technology are calculating correctly.
- When using spreadsheet technology for analysis, multiple problems can develop as students create and enter formulas for calculations. Write sample formula entries (i.e., the credit card payoff formula $= (4500 * (0.18/12))/(1 - (1 + (0.18/12))^\wedge(-12 * 3))$ on the board for them to emulate to help prevent this. Note that many programs require a $*$ symbol to denote multiplication. Placing two variable (or cells) side by side may generate a REF error.

Instructional Tasks

Instructional Task 1 (MTR.6.1, MTR.7.1)

You and your spouse are working on reducing your debt. You recently received a raise that increases your take home pay by $600 per month. Your current debts (outside of your mortgage) are listed below.

Debts	Current Balance	Interest Rate	Monthly Payment
Credit Card 1	$3,789.37	24.99%	$50.00
Credit Card 2	$1,806.68	22.99%	$50.00
Student Loan	$24,867.24	3.4%	$200.00

Part A. Would this extra income allow you to pay off these debts in four years, assuming no additional debt is incurred?

Part B. Compare the impact of paying increased payments for all three debts simultaneously versus focusing all the additional income on the smallest debt first.

Part C. How quickly could the smallest credit card be paid off? How long until the next card is paid off (consider the current monthly payment for card 2 could be applied to card 1 once card 2 is paid off)?

Part D. How long until the student loan is paid off?

Part E. Describe the benefits of paying off smaller debts first.

Instructional Items

Instructional Item 1

Jacqueline has a car loan she wants to pay off early. The loan is at 2.99% APR and has a current balance of $17,653.28. She currently makes a car payment of $387 each month. After paying off two of her credit cards, Jacqueline determines she could pay an additional $150 per month toward the car. Will this increased monthly payment allow her to pay off the car in 3 years? Why or why not?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.4 *Describe the advantages and disadvantages of financial and investment plans, including insurances*

MA.912.FL.4.1

Benchmark

MA.912.FL.4.1	Calculate and compare options, deductibles and fees for various types of insurance policies using spreadsheets and other technology.

Benchmark Clarifications:

Clarification 1: Insurances include medical, car, homeowners, life and rental car.

Clarification 2: Instruction includes types of insurance for a business and for an individual.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.1.1, MA.912.FL.1.2
- MA.912.FL.2.2, MA.912.FL.2.5, MA.912.FL.2.6

Terms from the K-12 Glossary

- Rate

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.2

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students analyze options for different types of insurance such as medical, car, homeowner's and life.

- **Auto Insurance**
  There are different factors that can affect an insurance premium, such as deductible, vehicle, mileage, driving history and personal information (i.e., age, sex, etc.). Students should consider this when comparing different options. There are also different requirements by state. Students should be able to identify the coverage needed based on the state where the car will be used (*MTR.7.1*).
  - Instruction includes calculating deductibles for each type of car insurance to determine the coverage needed. Students should also be able to determine when the deductible is applied and why (*MTR.4.1*).

Coverages	What is covered

Bodily injury liability (BI)	If driver is in an accident, this covers the bodily injury of anyone injured due to the accident.
Property damage liability (PD)	If driver is in an accident, this covers the damages caused to other's property.
Uninsured/underinsured motorist protection (UMP)	If driver is in an accident and the other driver is at fault, but does not have insurance, this covers all medical expenses for driver and passengers in one's car.
Personal injury protection (PIP)	This is also referred to as no-fault insurance and may be mandatory in different states. PIP covers all physical injuries for driver and passengers that could occur regardless of actual accident.
Comprehensive insurance	Coverage includes repair or replacement of car due to any damage, such as vandalism, fire, damage from tries, etc.
Collision insurance	If driver is in an accident, this covers repair or replacement of vehicle regardless of who is at fault. This may not a required coverage based on if the vehicle is financed or owned.
Car-rental insurance	Coverage will cover the cost of a car-rental if driver's vehicle is being repaired.
Emergency road service insurance	This coverage pays for any towing or road service. Some items, such as gas, oil, may not be covered.

  - For example, using situations where there is damage to the car, calculate the deductible (technology: calculator). Sam is in a car accident and there is $5,600 worth of damage to his car. His deductible is $1,000. Explain how much he would pay versus how much his insurance will pay (*MTR.5.1*).
  - For example, using situations where there was property damage, calculate the deductible (technology: calculator). Billy's car insurance policy covers $25,000 for personal property liability. He is involved in a car accident. There is $7,000 damage caused to the other vehicle and $2,300 damage caused to ancillary items, such as a mailbox or yard items. How much does Billy have to pay out of pocket to cover the damages?
  - For example, using situations where there were injuries to passengers and damage to property (technology: calculator). Sidney has a policy which covers $50,000 PD, $50,000 PIP and 50/150 BI. She is involved in an accident with an SUV. There are 3 people injured in the SUV and 2 people injured in her car. The total medical bill for all involved is $43,788. How much will Sidney need to pay out of pocket?
- When analyzing auto insurance, instruction includes analyzing the relationships between coverage limits, deductibles and premiums. Develop understanding of the inverse relationship between the deductible and premium, as well as other factors that may contribute to the cost of insurance, including car rentals.
  - For example, analyze the two situations where the PIP and Collision coverage is the same, but the premium is different. (*MTR.5.1*)

Insurance A	Insurance B

Heather pays $350 a month for her car insurance, which includes the following: Deductible = $500 Coverages	Sue pays $276 a month for her car insurance, which includes the following: Deductible = $1,000 Coverages

Both were involved in a car accident where there was $490 worth of property damage and $600 worth of damage to the car, which insurance is a better option and why?

- Develop understanding of different options when paying premium.
  - For example, paying monthly may include a surcharge on the premium. If the premium is paid for the entire year, it is $1,400. If it is paid monthly, each month the insurer pays $125. Which is a better option and why?

- **Health Insurance**
  When analyzing health insurance, instruction will include the various costs associated with having health insurance and the different plans available, as well as out of pocket expenses and deductibles based on the premium (*MTR.7.1*).
  - For example, compare the different types of insurance and which is a better option based on a variety of situations:
    - Using the chart below, determine and explain which policy would be best for the scenarios described.
    - Scenario A
      Jose selects the Insurance A for himself. During the year, he visits six in-network doctors and two out-of network doctors. Did Jose select the correct insurance premium?
    - Scenario B
      If Sam needs to visit an out-of-network doctor 15 times in one year, which plan would be the most financially advantageous and why?
    - Scenario C
      Paul signed his family up for HMO +. At the end of the year, they had 12 in-network visits and 10 out-of-network visits. Was this the best choice for Paul? Explain why you made this choice.

Coverages/co-pay	**Insurance – HMO**	**Insurance – HMO +**	**Insurance – PPO**
Medical Single Coverage	$97 per month	$106 per month	$120 per month
Medical Family Coverage	$427 per month	$531 per month	$647 per month
Co-Pay	In network $40 Out of network $50	In network $20 Out of network $40	In and out of network $30

  - When analyzing health insurance, instruction includes the cost covered by employers versus the cost covered by employees. Students will also be able to determine how much is deducted from paychecks. Students can use either a calculator or spreadsheet to help find the post. When using a spreadsheet, students should be able to determine the formulas for each cell.
    - For example, in Selena's new job, the health insurance premium being offered is $627 per month. Her employer agrees to pay 80%. How much will be deducted monthly from Selena's paycheck to cover her portion of health insurance? How much would be deducted if she gets paid every other week?

        - Instruction includes additional charges and deductibles, such as hospital stay, emergency room fees, etc.
    - Students will be able to determine if vision and/or dental are needed coverages. Insurance is offered as three different categories: Medical, Dental and Vision. Employees are able to select if they want one or a combination of the three. When evaluating dental and vision, students should be able to analyze the options, similar to medical, and decide which the best option is.
    - When providing instruction on health insurance, students should be able to see the difference between a plan provided by an employer versus an individual plan. Instruction includes an analysis of the difference and similarities between insurance plans including deductibles and premiums.
- **Homeowner's Insurance**
  Students should be able to compare renters and homeowners' insurance and when each is needed.
    - **Renter's Insurance**
      Within this benchmark, students should understand why it would be important to have Renter's insurance. Present students with scenarios to determine the coverage needed when renting a property (*MTR.7.1*).
    - **Homeowner's Insurance**
      Students should know and understand the different coverages needed as a homeowner. Instruction includes what homeowner's insurance covers, such as interior damage, exterior damage, loss or damage of personal assets, and injuries that may arise on the property (*MTR.7.1*).

Coverage	**What is covered?**
Property Dwelling Other Structures Personal Property Loss of Use	This coverage is for any damage caused to the home by a fire, hurricane, hail, lightning or other disasters. Most policies do not cover damage caused by floods, earthquakes or regular wear and tear. It is important that the coverage is enough to repair or rebuild the damage.
Liability Personal Liability Medical Payments to Others	This cover protects the homeowner from any lawsuits for bodily injury or property damage that the owners or family cause to others.

    - Provide scenarios where students need to determine the amount of coverage needed based on the house and property values.
        - For example, the Johnson family purchases a home for $354,000. Based on this, how much coverage do they need to ensure they have on their home?
- **Life Insurance**
    - When analyzing life insurance, instruction includes knowing the differences between term- and whole-life plans, as well as determining the premiums and coverage and which is the best option.
    - Students should be able to identify the best life insurance based on their different phases of life. For example, as a college student versus as a married couple versus with a family.
    - Instruction should include plans that are provided by employers and individual plans. Students should be able to determine when additional insurance may be

needed and why.

- When using spreadsheet technology for analysis, multiple problems can develop as students create and enter formulas for calculations. Note that many programs require a * symbol to denote multiplication. Placing two variables (or cells) side by side may generate a REF error.

Common Misconceptions or Errors

- When comparing different insurances premiums, look for students who may select one as more financially beneficial without considering the deductible.
- When deciding which car insurance premiums, look for students who may select a premium that will not cover what they need based on different scenarios.
- When analyzing premiums and coverages, look for students who do not consider all the coverages and may select a cheaper option. Help students understand the importance of truly getting the coverage needed for life circumstances.
- Previously students have used spreadsheets and other technology to help calculate math problems, look for students who may input the information incorrectly into the spreadsheet and/or technology.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Joey is trying to decide which car insurance policy is the best option. He receives three quotes from different insurance companies (6-month premium).

Coverage	Limits of Protection	Premium Insurance 1	Premium Insurance 2	Premium Insurance 3
Bodily Injury Liability	100,000-300,000	86	151.13	75
Property Damage Liability	100,000	84	included	60.50
Medical Payments	5,000	22	14.73	8.50
Comprehensive		73 (500 deductible)	15.45 (1,000 deductible)	37.50 (500 deductible)
Collision		235 (500 deductible)	85.55 (500 deductible)	167.50 (500 deductible)

Using technology, determine which Insurance company would be the best option and explain why.

Instructional Task 2 (MTR.7.1)

Susan is renting her first apartment and is considering renter's insurance. She receives quotes from several companies, which she puts in a spreadsheet along with her assets.

Assets	**Total**	$9,550	
	Jewelry - $750	*Electronics* - $3,000	*Clothing* - $2,000
	Furniture - $2,000	*Other Assets* - $1,800	
Company	**A**	**B**	**C**
Monthly Premium	$22	$29	$32
Coverages	$5,000-$50,000	$15,000-$30,000	$15,000-$100,000

Deductible	$1000	$750	$750

Part A. Based on the numbers provided, which company would be the best option?

Part B. If there is a robbery, and all the jewelry is stolen, is it beneficial to submit an insurance claim or pay out of pocket?

Instructional Items

Instructional Item 1

When selecting a health insurance premium, based on the chart below, explain which plan would be the least favorable and explain why.

Plan	HMO	Silver +	Silver
Monthly Premiums	$641.27	$489.92	$520.53
Deductibles			
Individual Deductible	$2,550	$2,700	$2,000
Family Deductible	$5,100	$5,400	$4,000
Out of Pocket Maximums			
Individual Deductible	$4,850	$5,000	$7,900
Family	$9,700	$10,000	$15,800
Co-Payments			
PCP	0.1	0	$60
Specialist	0.1	0.2	$80
Co-Insurance	0.1	0.2	0.2
Emergency Room	0.1	0.2	$0
Estimated ER Costs Based on $2,000 Visit	$200	$400	$400
Estimated ER Costs Before Co-Insurance	$2,000	$2,000	$2,000

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.4.2

Benchmark

MA.912.FL.4.2 Compare the advantages and disadvantages for adding on a one-time warranty to a purchase using spreadsheets and other technology.

Example: VicTorrious is a graphic designer and needs to buy a new computer every 3 years. For every computer that VicTorrious buys, she does not add on the one-time warranty because she feels that the total cost of the added-on warranties will be more than the total cost of all repairs she expects to have.

Benchmark Clarifications:

Clarification 1: Warranties include protection plans from stores, car warranties and home protection plans.

Clarification 2: Instruction includes types of warranties for a business and for an individual.

Clarification 3: Instruction includes taking into consideration the risk of utilizing or not utilizing a onetime warranty on one or multiple purchases.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.1.1, MA.912.FL.1.2

Terms from the K-12 Glossary

- Rate

- MA.912.FL.2.1, MA.912.FL.2.2, MA.912.FL.2.5

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.2

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students determine the advantages and disadvantages of a one-time warranty when purchasing items (*MTR.7.1*).

- **Protection Plans**
  Instruction includes an analysis on protection plans offered from stores. Students should be able to determine the financial advantages and disadvantages of purchasing the extended plans. Instruction should allow students the opportunity to decide when purchasing a protection plan is beneficial.
    - For example, Bella purchases a laptop for $879. She is offered the extended warranty at $189 for 3 years. After having the laptop for 2 years, she notices that there is an issue with the keyboard. She takes it in for service and it is $60 per hour or covered under the warranty if applicable. Is it a good idea for Bella to purchase the warranty? Explain why or why not using calculations using technology.
- **Car Warranties**
  Students should be able to analyze car warranties to determine if purchasing extended warranties are beneficial or when it is a recommended purchase.
    - For example, if certain parts break down after the normal warranty expires. Based on what they have paid for the warranty, compare it to cost of the parts to determine the benefits. On the opposite side, what if the extended warranty is not purchased and the part or repair is paid out of pocket. Would it have been more advantageous to purchase the warranty?
- **Home Protection Plans**
    - Students should use reasonableness to determine the need for home protection plans. Instruction should include analyzing different appliances in the home and which ones should have protection plans and which ones should not.
        - For example, students should compare the prices of purchasing items as they break, repairing same item, or purchasing the warranty.
- Students should analyze the types of warranties for a business and for an individual. Instruction includes an analysis of warranties based on the item and environment.
    - For example, if a company purchases 100 laptops versus an individual purchase. Statistically speaking, there is a higher probability of a laptop having issues when there are more to purchase than an individual purchase. Students should be able to decide which items, such as laptops, a warranty would be needed on if it is a business purchase or an individual purchase.
- Instruction includes the risk of utilizing or not utilizing a one-time warranty on one or multiple purchases. When purchasing items that could have a warranty, students should be able to determine what the risks of having the warranty are.
    - For example, cellular phones may offer a warranty through their insurance policy.

If a claim is made, what were to happen if the cellular phone breaks again? Students should be able to determine the risks of having the warranty versus not having it.

Common Misconceptions or Errors

- When comparing the different warranties, look for students who may select one as more or less advantageous based on the stipulations and items.
- Previously students have used spreadsheets and other technology to help calculate math problems, look for students who may input the information incorrectly into the spreadsheet and/or technology.

Instructional Tasks

Instructional Task 1

Cassie is purchasing a used car with approximately 26,000 miles. The car's original warranty is three years/36,000 bumper to bumper and five years/60,000 for powertrain coverage. During the purchase, the sales person suggest that she purchase an extended warranty. The extended warranty would cost an additional $2,200 and offer a six-year/72,000-mile warranty. Based on Cassie's research, the car seems to have limited issues during the warranty. Cassie averages about 7,000 miles per year and plans to keep the car for at least 3 more years. Explain the advantages and disadvantages of purchasing the extended warranty and when it would be essential to have purchased it.

Instructional Items

Instructional Item 1

Juan purchased a new dishwasher when he remodeled his kitchen. The company provided a 1-year warranty with the purchase price. Three years later, the dishwasher stopped working and it was discovered that it needed a new piece. The piece costs $250 plus labor, which is $50 an hour with a 3-hour minimum. Based on this scenario, which warranty would have been the best option for Juan to add during the initial purchase?

a. $150 for 3 years beyond manufacturer's warranty with 50% reimbursement of parts and labor
b. $360 for 5 years beyond manufacturer's warranty with 75% reimbursement for parts and 100% labor
c. $540 for 5 years beyond manufacturer's warranty with 100% reimbursement for parts and labor
d. No additional warranty

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.4.3

Benchmark

MA.912.FL.4.3	Compare the advantages and disadvantages of various retirement savings plans using spreadsheets and other technology.

Benchmark Clarifications:
Clarification 1: Instruction includes weighing options based on salary and retirement plans from different potential employers.
Clarification 2: Instruction includes the understanding the need to build one's own retirement plan when starting a business.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.2.5

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.2

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students determine which retirement savings plan would be most advantageous (*MTR.4.1*).

- Students should have an understanding of the different types of retirement savings plans, such as pension plans and 401ks. Students should also explore additional retirement options, such as 403b and Individual Retirement Accounts (IRAs). Instruction includes the advantages and disadvantages of each type of plan.
- Instruction includes analyzing different employers and deciding the best option based on salary and retirement plans.
  - For example, provide situations where both options (pension vs. 401k, both with employer contributions and without) are available and the salary is different. Students should be able to compare and justify the long-term advantages and disadvantages of both.
- Using technology, such as a spreadsheet, students should be able to calculate the long-term contributions given a variety of scenarios.
- Students understand the importance of having a retirement plan when self-employed. Students should compare the advantages and disadvantages of Traditional or Roth IRAs, Solo 401k, Self-employed people (SEP) IRA, Simple IRA or Defined Benefit Plan.
  - For example, provide examples showing the long-term effects of having a retirement savings plan versus not having one. What are the financial repercussions?

Common Misconceptions or Errors

- Students may misinterpret the information and determine that one employer is a better option; however, in the long run one would have to work longer.
- Students may input data incorrectly into the spreadsheet and not see the overall contributions to a retirement account.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

Jake just finished his degree and is applying for jobs. He receives three different offers and is looking at the retirement plans as one of the deciding factors. Based on the following chart, explain the advantages and disadvantages of each. Then explain which one he should choose and why.

Company	**A**	**B**	**C**
Annual Salary	$60,000	$62,000	$50,000
Retirement Option	401k – 100% Match	401k – 50% Match	Pension
Annual Employee Contribution	$5,000	$5,000	$0
Annual Employer Contribution	$5,000	$2,500	$10,000

Instructional Items

Instructional Item

John has decided to start his own business and wants to start his retirement account. Select the best option based on this salary and explain why.

	Amount
Annual Sales	$125,000
Annual Expenditures	$67,000
Additional expenses	$5,500

a. Traditional IRA - $6000 per year maximum
b. Solo 401k - $19,500 per year maximum
c. SEP IRA - $31,250
d. Simple IRA - $14,000

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.4.4

Benchmark

MA.912.FL.4.4 Collect, organize and interpret data to determine an effective retirement savings plan to meet personal financial goals using spreadsheets and other technology.

Example: Investigate historical rates of return for stocks, bonds, savings accounts, mutual funds, as well as the relative risks for each type of investment. Organize your results in a table showing the relative returns and risks of each type of investment over short and long terms, and use these data to determine a combination of investments suitable for building a retirement account sufficient to meet anticipated financial needs.

Benchmark Clarifications:

Clarification 1: Instruction includes students researching the latest information on different retirement options.

Clarification 2: Instruction includes the understanding of the relationship between salaries and retirement plans.

Clarification 3: Instruction includes retirement plans from the perspective of a business and of an individual.

Clarification 4: Instruction includes the comparison of different types of retirement plans, including IRAs, pensions and annuities.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.4.3

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.2

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students will research current data to explore different retirement plans and make decisions on the most effective plan based on given situations.

- Instruction includes understanding the latest information on the different retirement plans available, such as IRAs, 401k, Pension, Social Security and 403b. Students should be able to compare the different plans and analyze the benefits of each.
- Students should be able to use a spreadsheet to show the different plans available from different employers to justify their decision.
- **Salaries and Retirement Plans**
  Students should use technology to compare salaries and retirement plans in order to make the best overall financial decision.
  - For example, students should have examples for each of the options: IRAs, 401k, Pension, Social Security and 403b. Using this information and technology, students will be able to see which option would benefit them the most in the long run to ensure they are prepared for retirement and at what age.
- **Business versus Individual Plans**
  Students should understand how the retirement plans from both the individual and business perspective.
  - For example, provide students with a sample where the cost of the individual and the cost of the business is similar. Students should be able to determine which plan would be most beneficial for them based on their goals?
- **Retirement Plans**
  Students should be able to compare the different types of retirement plans: IRAs, pension plans and annuities. Based on this information, students will be able to know which type to request and at what percentage.
  - Instruction helps students determine when it is necessary to have one or all of the plans and why.

Common Misconceptions or Errors

- Students may misinterpret the information provided and determine a retirement plan that is not the most advantageous for employees.
- Students may not input the formulas into the spreadsheet correctly and generate an incorrect amount when analyzing the different plans.

Instructional Tasks

Instructional Task 1 (MTR.7.1)

Robert is looking for his first job. He is offered a starting salary of $45,000 with a 50% match on his 401k contributions. Based on this information, research the following.

Part A. How much should Robert contribute with each paycheck and why?

Part B. What additional retirement accounts should Robert consider? How much should he contribute? Explain.

Instructional Items

Instructional Item 1

Monica is starting a new job and the company has given her several options for retirement.

Option 1. Starting salary of $45,000; Employer retirement contributions: 5% match during the first year, 20% match up to year 10, then 100% match beyond year 10

Option 2. Starting Salary of $37,000; Employer retirement contributions: 100% match

Option 3. Starting Salary of $55,000; Employer retirement contributions: 50% match

Option 4. Starting Salary of $70,000; Employer retirement contributions: 0%

Which is the best option for Monica and why?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.4.5

Benchmark

MA.912.FL.4.5 Compare different ways that portfolios can be diversified in investments.

Benchmark Clarifications:

Clarification 1: Instruction includes diversifying a portfolio with different types of stock and diversifying a portfolio by including both stocks and bonds.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.2.3
- MA.912.FL.3.1
- MA.912.FL.4.4

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.2

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students explore options to make money through investments, including portfolios.

- Instruction ensures student understanding of what stocks and bonds are and the advantages/disadvantages of both. With this information, students should be able to understand what it means to diversify a portfolio and create a sample. Instruction focuses on the best stocks and bonds to invest in to ensure a profit was made, while building knowledge around risks and rewards (*MTR.7.1*).

- Students should understand the stock market and the different types of stocks available through the New York Stock Exchange (NYSE) and the National Association of Securities Dealers Automated Quotation System (NASDAQ). When learning about the stock market, students should understand how shares are associated with different companies. When investing, this means that the investor is part-owner of the company, while the company gets most of the capital. The investor may earn money in two ways: when trading stocks in the market or earning dividends from the company's profit. It is important for students to understand that some stocks are riskier than others and could result in a loss.
- Students should understand the basics of a bond and the different options available, corporate and government. Investing in a bond means one is lending the government or corporation money. Instruction includes the risks involved with purchasing bonds and help students identify the face value, maturity and coupon rate of a bond.
- Instruction includes the different levels of diversification: advanced, intermediate and basic. Instruction includes the differences between stocks and bonds and how they support one's portfolio. Along with understanding the differences, students should be able to determine which a better option is for themselves based on a variety of factors, specifically risk versus reward.
    - For example, what types of stocks should be in one's portfolio and why? What is the ideal combination of stocks and bonds for a diversified portfolio? Is it necessary to have both in a portfolio?

Common Misconceptions or Errors

- Students may not diversify their portfolio to have a balance of both stocks and bonds. Be cautious of students who may only include all stocks or all bonds. It is important for students to understand risks associated with stocks.

Instructional Tasks

Instructional Task 1 (MTR.4.1)

Beth is starting her portfolio and has $40,000 to invest. Identify at least 3 stocks or bonds that she should include in her portfolio. Explain why these would be the best option for her.

Instructional Items

Instructional Item 1

Alex is looking to diversity his portfolio. Select which level of diversification (basic, intermediate or advanced) would minimize his risk and explain why.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.FL.4.6

Benchmark

MA.912.FL.4.6	Simulate the purchase of a stock portfolio with a set amount of money, and evaluate its worth over time considering gains, losses and selling, taking into account any associated fees.

Connecting Benchmarks/Horizontal Alignment

- MA.912.FL.1.1
- MA.912.FL.1.2
- MA.912.FL.4.5

Terms from the K-12 Glossary

Vertical Alignment

Previous Benchmarks

- MA.7.AR.3.2

Next Benchmarks

Purpose and Instructional Strategies

In Math for Data and Financial Literacy, students create their own portfolios and calculate their profits and losses (*MTR.5.1, MTR.7.1*).

- Students should work with real-life scenarios that simulates the creation of a stock portfolio within their budget. Considerations that a simulation includes are guidelines on how to monitor their portfolio and when to sell, buy or trade within a given timeline; use of technology to calculate gains, losses and any fees based on sales; and monitor portfolio based on given time frame.
- Instruction includes how to calculate stock profit using technology and the formulas below.
    - $Total\ Buy\ Price = shares \times\ buy\ price + commissions$
    - $Total\ Sell\ Price = shares \times sell\ price + commission$
    - $Total\ Profit\ or\ Loss = Total\ Buy\ Price - Total\ Sell\ Price$
- Students should be able to explain why they made their selections and what they would alter during another simulation.

Common Misconceptions or Errors

- Students may not make the best decisions when diversifying simulated portfolio.
- Students may incorrectly calculate their gains and/or losses based on when the stock is bought or traded.
- Students may buy or sell their stock and/or bond at a time that is not beneficial.
- Students may not include dividends when it is offered by the company. Remind students to research this as part of their development of their portfolio.

Instructional Tasks

Instructional Task 1 (MTR.5.1, MTR.7.1)

Billy has saved up $30,000 to create a portfolio. Identify stocks and/or bonds to help diversify his portfolio. Then monitor the portfolio over a 6-week period, determine when it is a good time to sell or buy more, and determine his profits and/or losses and the end of the time period. Use technology to monitor the portfolio and explain what strategies were effective and ineffective.

Instructional Items

Instructional Item 1

Jan purchased 150 shares for $1,018.50 in a video game stock 5 years ago.

Share price (5 years ago): $6.79

Share price (today): $354.44

How much is Jan's profit after the 5 years if she sells the 100 shares?

a. $35,444.00
b. $34,765.00
c. $52,147.50
d. $53,166.00

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.1 *Summarize, represent and interpret categorical and numerical data with one and two variables.*

MA.912.DP.1.2

Benchmark

MA.912.DP.1.2 Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether its univariate or bivariate and interpret the different components and quantities in the display.

Benchmark Clarifications:
Clarification 1: Within the Probability and Statistics course, instruction includes the use of spreadsheets and technology.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.2.4, MA.912.DP.2.8, MA.912.DP.2.9
- MA.912.DP.3.1, MA.912.DP.3.2, MA.912.DP.3.3
- MA.912.DP.5.11

Terms from the K-12 Glossary

- Bivariate data
- Categorical data
- Data

Vertical Alignment

Previous Benchmarks

- MA.912.DP.1.1

Next Benchmarks

- MA.912.DP.1.3, MA.912.DP.1.4, MA.912.DP.1.5

Purpose and Instructional Strategies

In Algebra I, students selected appropriate methods to represent categorical or numerical data, whether univariate or bivariate. In Mathematics for Data and Financial Literacy Honors, students interpret these data distributions as it relates to various types of real-world data. Students determine whether the data is numerical or categorical, whether it's univariate or bivariate, and interpret the different components in the representation. In other courses, students will interpret data distributions using spreadsheets and technology, explain the difference between correlation and causation, interpret the margin of error of a mean or percentage from a data set, and interpret the confidence level corresponding to the margin of error.

- Instruction includes interpreting data sets for numerical and categorical data whether univariate or bivariate.
    - For numerical univariate data, representations include histograms, stem-and-leaf plots, box plots and line plots.
    - For numerical bivariate data, representations include scatterplots and line graphs.
    - For categorical univariate data, representations include bar charts, circle graphs, line plots, frequency tables and relative frequency tables.
    - For categorical bivariate data, representations include segmented bar graphs, joint frequency tables and joint relative frequency tables.
- It is the intention of this benchmark to include cases where students must calculate measures of center/variation to interpret *(MTR.3.1)*.

- Instruction makes the connection to relevant, real-world applications such as discretionary expenses, consumer credit, automobile ownership, employment, taxes, independent living, the stock market, business relations, planning for retirement, budgeting, current events, etc. *(MTR.7.1)*.
- This benchmark reinforces the importance of the use of questioning within instruction *(MTR.4.1)*.
    - Does this display univariate or bivariate data?
    - Is the data numerical or categorical?
    - What do the different quantities within the data display mean in terms of the context of the situational data?

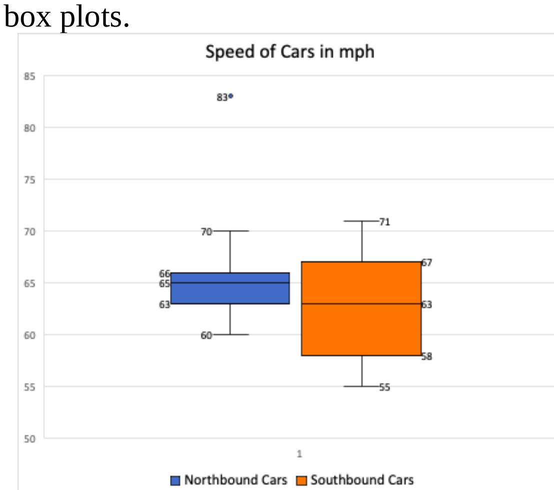
Common Misconceptions or Errors

- Students may not be able to properly distinguish between numerical and categorical data, or between univariate and bivariate date.
- Students may misidentify and/or misinterpret the quantities in the various data displays.
- Students may not be able to distinguish between the measures of center and the measures of spread.
- Students may not completely grasp the effect of outliers on the data set, or incorrectly conclude a point is an outlier.
- Students may not be able to distinguish the differences between frequencies and relative frequencies, or identify the condition that determines a conditional or relative frequency in a joint table.

Instructional Tasks

Instructional Task 1 (MTR.3.1, MTR.4.1, MTR.7.1)

A statistically minded state trooper wondered if the speed distributions are similar for cars traveling northbound and for cars traveling southbound on an isolated stretch of interstate highway. He uses a radar gun to measure the speed of all northbound cars and all southbound cars passing a particular location during a 15-minute period. Below are his results represented in a set of box plots.

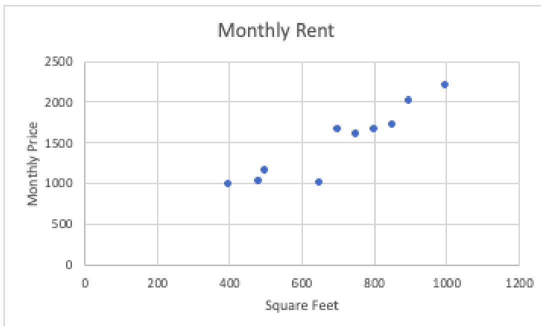


Part A. Is the data displayed above numerical or categorical?
Part B. Does the display represent univariate or bivariate data?

Part C. Using the box plots above, compare the data of the northbound cars with the data from the southbound cars. Make sure to include measures of center and measures of spread in your response.
Part D. In what ways are box plots a good representation of the data for the purpose of comparing?
Part E. What would be another appropriate way to represent the data for the purpose of comparing? In what ways would that representation help to better compare the data than box plots?

Instructional Task 2 (MTR.5.1)

While searching for apartments, Jaden wanted to explore the correlation between the price and the amount of square feet of the apartment. The scatterplot below shows Jaden's findings.

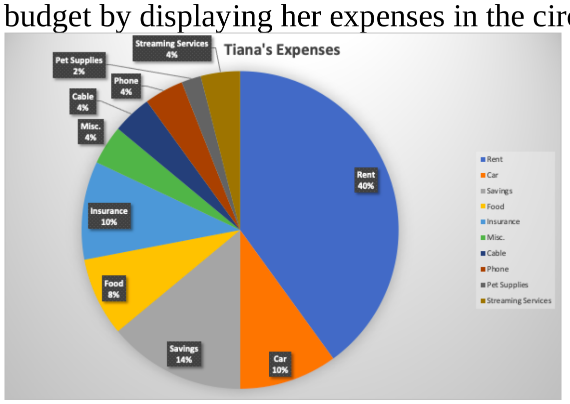


Part A. Is the data displayed in the scatterplot numerical or categorical?
Part B. Does the display represent univariate or bivariate data?
Part C. Does there appear to be a positive, negative or no correlation between the price and the amount of square feet of the apartment?

Instructional Items

Instructional Item 1

Tiana made a personal budget by displaying her expenses in the circle graph below.



Part A. Is the data displayed in the circle graph numerical or categorical?
Part B. Does the display represent univariate or bivariate data?
Part C. What statement(s) can be made related to Tiana's expenses and her savings per month?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.2 *Solve problems involving univariate and bivariate numerical data.*

MA.912.DP.2.4

Benchmark

MA.912.DP.2.4 Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.

Benchmark Clarifications:

Clarification 1: Instruction includes fitting a linear function both informally and formally with the use of technology.

Clarification 2: Problems include making a prediction or extrapolation, inside and outside the range of the data, based on the equation of the line of fit.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.1.2
- MA.912.DP.2.8
- MA.912.DP.2.9
- MA.912.DP.5.11

Terms from the K-12 Glossary

- Bivariate data
- Cluster (data)
- Intercept
- Line of fit
- Linear function
- Scatterplot
- Slope
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.8.DP.1.1
- MA.8.DP.1.2
- MA.8.DP.1.3
- MA.912.DP.1.1

Next Benchmarks

- MA.912.DP.2.7

Purpose and Instructional Strategies

In grade 8, students worked with scatterplots and lines of fit. In Math for Data and Financial Literacy, students relate the slope and y-intercept of a line of fit to association in bivariate numerical data and interpret these features in real-world contexts.

- This is an extension of MA.912.DP.1.2, where students interpret numerical bivariate data. It is good to review with students that a scatterplot is a display of numerical bivariate data sets. Benefits of scatter plots are listed below.
    - Scatter plots can show a relationship or association between two variables.
    - They can reveal trend lines or shapes of trends that can be used to predict or estimate values.
    - They are useful for highlighting outliers within the data sets.
- In this benchmark, students are fitting a linear function to numerical bivariate data, interpreting the slope and y-intercept based on the context and using that linear function to make predictions about values that correspond to parts of the graph that lie beyond or within the scatter plot.
- Instruction makes the connection to relevant, real-world applications such as discretionary expenses, consumer credit, automobile ownership, employment, taxes, independent living, the stock market, business relations, planning for retirement,

budgeting, current events, etc. *(MTR.7.1).*

- Instruction includes the use of technology for students to understand the difference between a line of fit and a line of best fit. Additionally, instruction of this benchmark should lead into non-linear models such as quadratic models (MA.912.DP.2.8) and exponential models (MA.912.DP.2.9).
- During instruction it is important to distinguish the difference between a "line of fit" and the "line of best fit."
    - A "line of fit" is used when students are visually investigating numerical bivariate data that appears to have a linear relationship and can sketch a line (using a writing instrument and straightedge) that appears to "fit" the data. Using this "line of fit" students can estimate its slope and y-intercept and use that information to interpret the context of the data.
    - The "line of best fit" (also referred to as a "trend line") is used when the data is further analyzed using linear regression calculations (the process of minimizing the sum of the squared distances from the individual data values to the line), often done with the assistance of technology.

Common Misconceptions or Errors

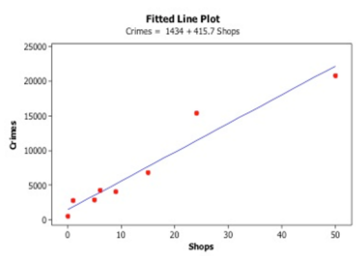
- Students may not know how to sketch a line of fit.
    - For example, they may always go through the first and last points of data.
- Students may confuse the two variables when interpreting the data as related to the context.
- Students may not know the difference between interpolation (predictions within a data set) and extrapolation (predictions beyond a data set).
- Students may need assistance working with spreadsheets, calculator, or other types of technology as it relates to data.

Instructional Tasks

Instructional Task 1

Many counties in the United States are governed by a county council. At public county council meetings, county residents are usually allowed to bring up issues of concern. At a recent public County Council meeting, one resident expressed concern that 3 new coffee shops from a popular coffee shop chain were planning to open in the county, and the resident believed that this would create an increase in property crimes in the county. (Property crimes include burglary, larceny-theft, motor vehicle theft, and arson.) To support this claim, the resident presented the following data and scatterplot (with the least-squares line shown) for 8 counties in the state.

County	Shops	Crimes
A	9	4000
B	1	2700
C	0	500
D	6	4200
E	15	6800
F	50	20800
G	5	2800
H	24	15400



The scatterplot shows a positive linear relationship between “Shops” (the number of coffee shops of this coffee shop chain in the county) and “Crimes” (the number of annual property crimes for the county). In other words, counties with more of these coffee shops tend to have more property crimes annually.

Part A. Does the relationship between Shops and Crimes appear to be linear? Would you consider the relationship between Shops and Crimes to be strong, moderate, or weak?

Part B. The equation of the least-squares line for these data is $C = 1434 + 415.7s$, where C represents the number of predicted annual property crimes and s represent the number of coffee shops. Based on this line, what is the estimated number of additional annual property crimes for a given county that has 3 more coffee shops than another county?

Part C. Do these data support the claim that building 3 additional coffee shops will necessarily *cause* an increase in property crimes? What other variables might explain the positive relationship between the number of coffee shops for this coffee shop chain and the number of annual property crimes for these counties?

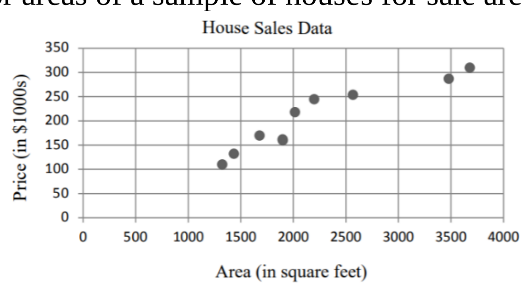
Part D. If the following two counties were added to the data set, would you still consider using a line to model the relationship? If not, what other types (forms) of model would you consider?

I	25	36,900
J	27	24,100

Instructional Items

Instructional Item 1

The prices and total floor areas of a sample of houses for sale are shown in the scatterplot.



Part A. Draw a line that appears to be a good fit for the data on the graph.
Part B. Write the equation of your line of fit.
Part C. Use your equation to predict the price of a 3,000 square foot house. Show all work.

MA.912.DP.2.8

Benchmark

MA.912.DP.2.8	Fit a quadratic function to bivariate numerical data that suggests a quadratic association and interpret any intercepts or the vertex of the model. Use the model to solve real-world problems in terms of the context of the data.

Benchmark Clarifications:
Clarification 1: Problems include making a prediction or extrapolation, inside and outside the range of the data, based on the equation of the line of fit.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.1.2
- MA.912.DP.2.4, MA.912.DP.2.9
- MA.912.DP.5.11

Terms from the K-12 Glossary

- Bivariate data
- Cluster (data)
- Line of fit
- Quadratic function
- Scatterplot
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks

- MA.8.DP.1.1, MA.8.DP.1.2, MA.8.DP.1.3
- MA.912.DP.1.1

Next Benchmarks

- MA.912.DP.2.7

Purpose and Instructional Strategies

In grade 8, students work with scatterplots and lines of fit. In Math for Data and Financial Literacy, students work with bivariate numerical data that suggests a quadratic association and interpret the intercepts or the vertex of the model as it relates to a real-world context.

- In this benchmark, students are fitting a quadratic function to numerical bivariate data, interpreting the intercepts and vertex based on the context and using that quadratic function to make predictions about values that correspond to parts of the graph that lie beyond or within the scatter plot.
- Instruction makes the connection to relevant, real-world applications such as discretionary expenses, consumer credit, automobile ownership, employment, taxes, independent living, the stock market, business relations, planning for retirement, budgeting, current events, etc. *(MTR.7.1)*.
- Instruction includes the use of technology for students to understand the difference between a curve of fit and a curve of best fit.
- During instruction it is important to distinguish the difference between a "curve of fit" and the "curve of best fit."
  - A "curve of fit" is used when students are visually investigating numerical bivariate data that appears to have a non-linear relationship and can sketch a line or curve (using a writing instrument and straightedge) that appears to "fit" the data. Using this "curve of fit" students can estimate its vertex and intercepts and use that information to interpret the context of the data.
  - The "curve of best fit" is used when the data is further analyzed using regression calculations (the process of minimizing the squared distances from the individual data values to the line), often done with the assistance of technology.

Common Misconceptions or Errors

- Students may not know how to sketch a curve of fit.
    - For example, they may always go through the first and last points of data.
- Students may confuse the two variables when interpreting the data as related to the context.
- Students may have trouble interpreting the vertex and intercepts in relation to the context of the problem.
- Students may not know the difference between interpolation (predictions within a data set) and extrapolation (predictions beyond a data set).
- Students may need assistance working with spreadsheets, calculators or other types of technology as it relates to data.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

A study was done to compare the speed x (in miles per hour) with the mileage y (in miles per gallon) of a car with a specific make and model. The results are shown in the table below.

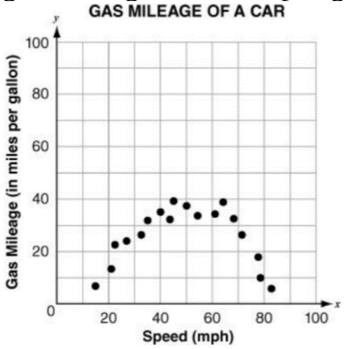
x	y
15	21.5
25	26.8
35	29.2
45	35
50	28.7
55	26.2
65	22.3
75	20

Part. A. Create a scatterplot of the data.
Part B. Sketch a line of fit for the data. Interpret the vertex and any intercepts in the context of the data.
Part C. Using your line of fit, estimate the mileage when the car is traveling at a speed of 60 miles per hour.
Part D. Use technology to find a model that best fits the data.
Part E. Use the model to predict the mileage of the car when the car is traveling at a speed of 60 miles per hour. How close was your prediction to the model's prediction?

Instructional Items

Instructional Item 1

The graph below represents the gas mileage, in miles per gallon, of a car at different speeds.



If x denotes the speed and y denotes the mileage per gallon, which equation best represents the relationship between the speed of the car and fuel efficiency?

a. $y = 0.025x^2 - 2.5x - 25$
b. $y = 0.025x^2 + 2.5x + 25$
c. $y = -0.025x^2 + 2.5x + 25$
d. $y = -0.025x^2 + 2.5x - 25$

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.2.9

Benchmark

MA.912.DP.2.9	Fit an exponential function to bivariate numerical data that suggests an exponential association. Use the model to solve real-world problems in terms of the context of the data.

Benchmark Clarifications:

Clarification 1: Instruction focuses on determining whether an exponential model is appropriate by taking the logarithm of the dependent variable using spreadsheets and other technology.

Clarification 2: Instruction includes determining whether the transformed scatterplot has an appropriate line of best fit, and interpreting the y-intercept and slope of the line of best fit.

Clarification 3: Problems include making a prediction or extrapolation, inside and outside the range of the data, based on the equation of the line of fit.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.1.2
- MA.912.DP.2.4, MA.912.DP.2.8
- MA.912.DP.5.11

Terms from the K-12 Glossary

- Bivariate data
- Exponential function
- Line of fit
- Scatterplot
- Slope
- x-intercept
- y-intercept

Vertical Alignment

Previous Benchmarks
- MA.8.DP.1.1, MA.8.DP.1.2, MA.8.DP.1.3
- MA.912.DP.1.1

Next Benchmarks
- MA.912.DP.2.7

Purpose and Instructional Strategies

In grade 8, students work with scatter plots and lines of fit. In Math for Data and Financial Literacy, students work with bivariate numerical data that suggests an exponential association and use the model as it relates to a real-world context.

- Instruction focuses on determining whether an exponential model is appropriate by taking the logarithm of the dependent variable using spreadsheets, calculators, or other technology. When plotting the data with the $log(dependent\ variable)$ as the vertical axis and the independent variable as the horizontal axis, if the resulting display is a straight line, then the relationship between the independent variable and dependent variable is exponential.
- Instruction includes interpreting the y-intercept and the slope of the transformed function.
    - Transforming to a linear model using logarithms $y = a(b)^x$, where a is the initial amount and b is the growth factor.
    $$\log(y) = \log(a(b)^x)$$
    $$\log(y) = \log a + \log b^x$$
    Transformed model: $\log(y) = x \log b + \log a$, with $\log b$ representing the rate of change and $\log a$ representing the initial value.
- Instruction makes the connection to relevant, real-world applications such as discretionary expenses, consumer credit, automobile ownership, employment, taxes, independent living, the stock market, business relations, planning for retirement, budgeting, current events, etc. *(MTR.7.1)*.
- Instruction includes the use of technology for students to understand the difference between a curve of fit and a curve of best fit.
- During instruction it is important to distinguish the difference between a "curve of fit" and the "curve of best fit."
    - A "curve of fit" is used when students are visually investigating numerical bivariate data that appears to have a non-linear relationship and can sketch a line or curve (using a writing instrument and straightedge) that appears to "fit" the data. Using this "curve of fit" students can estimate its vertex and intercepts and use that information to interpret the context of the data.
    - The "curve of best fit" is used when the data is further analyzed using regression calculations (the process of minimizing the squared distances from the individual data values to the line), often done with the assistance of technology.

Common Misconceptions or Errors

- Students may not know how to sketch a curve of fit.
    - For example, they may always go through the first and last points of data.
- Students may confuse the two variables when interpreting the data as related to the context.
- Students may have trouble transforming the exponential relationship to a linear one using logarithms.

- Students may not know the difference between interpolation (predictions within a data set) and extrapolation (predictions beyond a data set).
- Students may need assistance working with spreadsheets, calculators or other types of technology as it relates to data.

Instructional Tasks

Instructional Task 1 (MTR.7.1, MTR.2.1, MTR.4.1, MTR.5.1)

Pharmaceutical company A is developing a model to determine the length of time it takes a person's body to absorb a new drug being tested. The drug is administered to the subject (800mg) and a blood sample is taken from the subject every three hour. The table below shows the results.

Drug Absorption	
Hours Since Drug Administered	**Amount of Drug in Body (mg)**
0	800
3	630
6	410
9	342
12	225
15	165
18	104
21	73
24	49
27	38
30	27

Part A. Is an exponential relationship appropriate to model the data? Explain.
Part B. Create a scatter plot to model the data by transforming the data to a linear model.
Part C. Sketch a line or curve of fit for the transformed model.
Part D. Interpret the slope and y-intercept of the transformed model in terms of the context.
Part E. If a drug is not detectable when there is less than 0.005 mg in a person's bloodstream, estimate how long it would take for the drug to not be detected.

Instructional Items

Instructional Item 1

The table below gives air pressures in kPa at selected altitudes measured in km.

x (km)	0	1	2	3	4	5
y (kPa)	99	89	80	73	65	60

Part A. Use technology to determine a model that best fits the data.
Part B. Use the model from Part A to estimate the air pressure in kPa 10 km above sea level.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.3 *Solve problems involving categorical data.*

MA.912.DP.3.1

Benchmark

MA.912.DP.3.1 Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.

Algebra I Example: Complete the frequency table below.

	Has an A in math	Doesn't have an A in math	Total
Plays an instrument	20		90
Doesn't play an instrument	20		
Total			350

Using the information in the table, it is possible to determine that the second column contains the numbers 70 and 240. This means that there are 70 students who play an instrument but do not have an A in math and the total number of students who play an instrument is 90. The ratio of the joint frequencies in the first column is 1 to 1 and the ratio in the second column is 7 to 24, indicating a strong positive association between playing an instrument and getting an A in math.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.3.2
- MA.912.DP.3.3
- MA.912.DP.3.4
- MA.912.DP.5.11

Terms from the K-12 Glossary

- Bivariate data
- Categorical data
- Frequency table
- Joint frequency

Vertical Alignment

Previous Benchmarks

- MA.7.DP.1
- MA.8.DP.1
- MA.912.DP.1.1

Next Benchmarks

- MA.912.DP.3.5
- MA.912.DP.4.5

Purpose and Instructional Strategies

In grades 7 and 8, students explored the relationship between experimental and theoretical probabilities. In Math for Data and Financial Literacy, students study bivariate categorical data and display it in tables showing joint frequencies and marginal frequencies and determine possible associations in terms of real-world contexts. In other courses, students will interpret the joint and marginal frequencies as empirical probabilities.

- Instruction includes the connection to MA.912.DP.1.1 where students work with categorical bivariate data and display it in tables. A two-way frequency table is just a way to display frequencies jointly for two categories. A two-way frequency table is a way to display frequencies jointly for two categories.
- In order to interpret the joint and marginal frequencies, students must know the difference

between the two.

  - Marginal frequencies guide students to understand that the total column and total row are in the "margins" of the table, thus they are referred to as the marginal frequencies.
  - Joint frequencies guide students to connect that the word joint refers to the coming together of more than one, therefore the term joint frequency refers to combination of two categories or conditions happening together.
- Once the two-way table is complete, students can compare two ratios to assess the association of the data.
  - When comparing joint frequencies, students can either compare the ratios of the two joint frequencies of each of the columns (as was done in the Benchmark Example) or they can compare the ratios of the two joint frequencies in each of the rows.
    - For example, the completed example from the standard is displayed below.

	Has an A in Math	**Doesn't have an A in Math**	**Total**
Plays an instrument	20	70	90
Doesn't play an instrument	20	240	260
Total	40	310	350

- Comparing the two joint frequencies of the rows we get a 2/7 or approximately 0.29 ratio for the 1st row and a 1/12 or approximately .08 ratio for the 2nd row. Because these ratios are not close to each other and the 1st row is higher, this indicates a strong positive association between playing an instrument and having an A in math. One could also state there is a strong negative association between doesn't play an instrument and having an A in math.
- Comparing marginal frequencies with joint frequencies has a similar process. The ratio of the joint frequency 'Has an A in math and plays an instrument' to the marginal frequency 'Has an A in math' is 20/40 or 0.5. The ratio of the joint frequency 'Doesn't have an A in math and plays an instrument' to the marginal frequency 'Doesn't have an A in math' is 70/310 or 0.23. Since these ratios are not close and the first one is higher, this implies a strong positive correlation between having an A in math and playing an instrument.

Common Misconceptions or Errors

- Students may not be able to properly complete the table based on the data given.
- Students may not be able to distinguish the differences between marginal and joint frequencies.
- Students may not be able to properly identify the relationships and possible associations in the data in terms of the context given.
- When determining association, students may not be able to assess whether the association is positive or negative since it is based on how you state your justification.

Instructional Tasks

Instructional Task 1

The table displays the results of a survey of eating preferences of a sample of high school students. Complete the data in the two-way frequency table below then answer the following questions.

	Vegetarian	**Not a Vegetarian**	**Total**
Male		72	
Female	63		100
Total			200

Part A. Interpret each of the joint frequencies in terms of the context.
Part B. Interpret each of the marginal frequencies in terms of the context.
Part C. What can be said about the association between being a male and being a vegetarian?

Instructional Items

Instructional Item 1

Complete the frequency table below based on data collected randomly from 500 people at the local grocery store.

	Prefers Brand A	**Prefers Brand B**	**Total**
Between Ages 18-35	110		
Between Ages 36-60		240	
Total	200		500

Is there an association between age and the brand someone prefers?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.3.2

Benchmark

MA.912.DP.3.2 Given marginal and conditional relative frequencies, construct a two-way relative frequency table summarizing categorical bivariate data.

Algebra I Example: A study shows that 9% of the population have diabetes and 91% do not. The study also shows that 95% of the people who do not have diabetes, test negative on a diabetes test while 80% who do have diabetes, test positive. Based on the given information, the following relative frequency table can be constructed.

	Positive	Negative	Total
Has diabetes	7.2%	1.8%	9%
Doesn't have diabetes	4.55%	86.45%	91%

Benchmark Clarifications:

Clarification 1: Construction includes cases where not all frequencies are given but enough are provided to be able to construct a two-way relative frequency table.

Clarification 2: Instruction includes the use of a tree diagram when calculating relative frequencies to construct tables.

Connecting Benchmarks/Horizontal Alignment

Terms from the K-12 Glossary

- MA.912.DP.3.1
- MA.912.DP.3.3
- MA.912.DP.3.4
- MA.912.DP.5.11

- bivariate data
- categorical data
- conditional relative frequency
- frequency table
- joint frequency
- joint relative frequency

Vertical Alignment

Previous Benchmarks

- MA.7.DP.1.3
- MA.7.DP.2
- MA.8.DP.2
- MA.912.DP.1.1

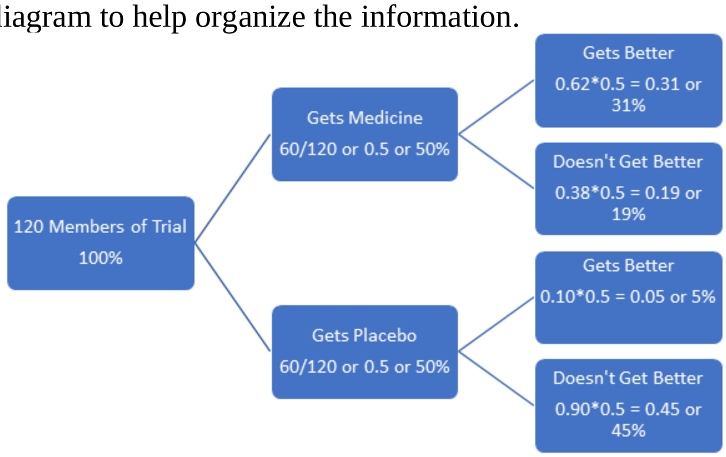
Next Benchmarks

- MA.912.DP.3.5
- MA.912.DP.4.5

Purpose and Instructional Strategies

In grades 7 and 8, students explored the relationship between experimental and theoretical probabilities. In Math for Data and Financial Literacy, students construct two-way relative frequency tables given marginal and conditional relative frequencies and summarize the data. In later courses, students will interpret the joint and marginal frequencies as empirical probabilities.

- Instruction includes the understanding that relative frequencies can be represented by a percentages or fractions.
- Instruction includes the connection to tree diagrams from determining sample spaces in repeated experiments from grade 8. Utilizing tree diagrams can help students to organize information while constructing relative frequency tables.
    - For example, a person takes part in a medical trial that tests the effect of a medicine on a disease. Half the people are given medicine and the other half are given a placebo, which has no effect on the disease. The medicine has a 62% chance of curing someone. But people who do not get the medicine still have a 10% chance of getting well. There are 120 people in the trial and they all have the disease. Construct a two-way relative frequency table to summarize the data. Use a tree diagram to help organize the information.



From this point, a student can complete a two-way relative frequency table to summarize the data.

	Gets Better	**Doesn't get Better**	**Total**
Gets Medicine	31%	19%	50%
Gets Placebo	5%	45%	50%

Total	36%	64%	100%

Common Misconceptions or Errors

- Students may not be able to place the information correctly in a tree diagram in order to find the missing information.
- Students may not perform the calculations correctly needed for the completion of the two-way frequency table.

Instructional Tasks

Instructional Task 1 (MTR.7.1, MTR.2.1)

A college student wondered if there was a correlation between a person working during the day or late-night and whether they drink coffee or not. He randomly surveyed students at his school. In his sample, 50% consisted of those who work during the day. The study showed that 76% of those who work at night surveyed were not coffee drinkers, while 63% of those who worked during the day were coffee drinkers.

Part A. Construct a tree diagram to organize the data.
Part B. Use the tree diagram you created to construct a two-way relative frequency table.
Part C. Summarize the joint and marginal frequencies represented in the table.

Instructional Items

Instructional Item 1

A study shows that 26.9% of the population drive a sports car and 73.1% do not. The study also shows that 78% of the people who drive a sports car, run on a regular basis while 85% who do not drive a sports, do not run on a regular basis. Based on the given information, construct a relative frequency table summarizing the data.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.3.3

Benchmark

MA.912.DP.3.3 Given a two-way relative frequency table or segmented bar graph summarizing categorical bivariate data, interpret joint, marginal and conditional relative frequencies in terms of a real-world context.

Algebra I Example: Given the relative frequency table below, the ratio of true positives to false positives can be determined as 7.2 to 4.55, which is about 3 to 2, meaning that a randomly selected person who tests positive for diabetes is about 50% more likely to have diabetes than not have it.

	Positive	Negative	Total
Has diabetes	7.2%	1.8%	9%
Doesn't have diabetes	4.55%	86.45%	91%

Benchmark Clarifications:
Clarification 1: Instruction includes problems involving false positive and false negatives.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.3.1
- MA.912.DP.3.2
- MA.912.DP.3.4
- MA.912.DP.5.11

Terms from the K-12 Glossary

- Bivariate data
- Categorical data
- Conditional relative frequency
- Frequency table
- Joint frequency
- Joint relative frequency

Vertical Alignment

Previous Benchmarks

- MA.7.DP.1.3
- MA.7.DP.2
- MA.8.DP.2
- MA.912.DP.1.1

Next Benchmarks

- MA.912.DP.3.5
- MA.912.DP.4.5

Purpose and Instructional Strategies

In grades 7 and 8, students explored the relationship between experimental and theoretical probabilities. In Math for Data and Financial Literacy, students discuss false positives and false negatives. In other courses, students will interpret the joint and marginal frequencies as empirical probabilities.

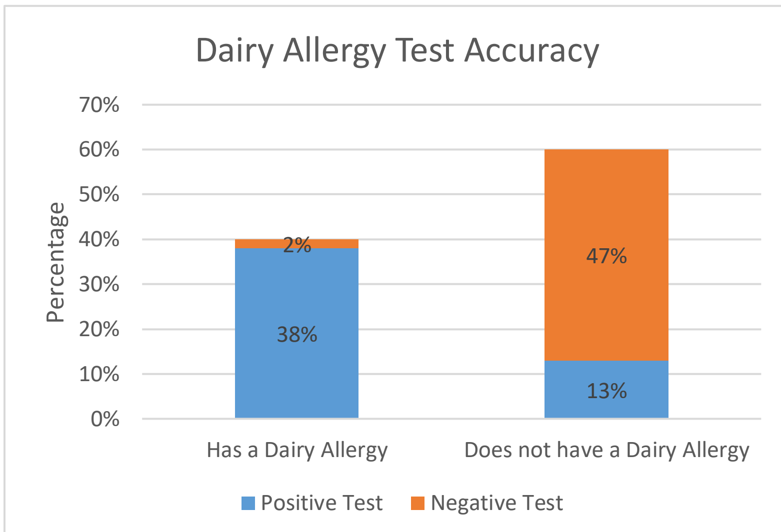
- In this benchmark, students will calculate and interpret joint, marginal and conditional relative frequencies in terms of a real-world context given a relative frequency table or a segmented bar graph.
- Instruction includes problems involving false positives and false negatives.
  - A true positive occurs when a model correctly identifies or predicts a positive outcome.
  - A false negative occurs when a model incorrectly identifies or predicts a positive outcome.

- A true negative occurs when a model correctly identifies or predicts a negative outcome.
- A false negative occurs when a model incorrectly identifies or predicts a negative outcome.

- False positives and false negatives are especially relevant when looking at medical studies.
    - For example, a study randomly tests 100 people for a dairy allergy. The two-way frequency table to represent this data is shown below.

	Positive Test	**Negative Test**	**Total**
Has a Dairy Allergy	38%	2%	40%
Does not have a Dairy Allergy	13%	47%	60%
Total	51%	49%	100%

The above example can also be represented in a segmented bar graph like the one below.



- To summarize the data, instruction includes interpreting joint, marginal, and conditional relative frequencies.
    - Joint Relative Frequency is the ratio of the frequency in a certain category to the total number of data points.
        - For example, the joint relative frequency of having a dairy allergy and testing positive is 38%, while the joint relative frequency of having a dairy allergy and testing negative is 2%. The ratio of true positives to false negatives is $\frac{38}{2}$ or 19. This means that a person who has a dairy allergy is 19 times more likely to have a true positive test than a false negative.
    - Marginal Relative Frequency is the ratio of the sum of the joint relative frequencies in a row or a column and the total number of data values.
        - The ratio of not having a dairy allergy to having a dairy allergy is $\frac{60}{40}$or $\frac{3}{2}$. This means participants were 50% more likely to not have a dairy allergy.
    - Conditional Relative Frequency is the ratio of a joint relative frequency a

related marginal relative frequency.

- For example, the conditional relative frequency of participants with a dairy allergy that tested positive is $\frac{38}{40}$or 95% while the conditional relative frequency of participants without a dairy allergy who tested positive is $\frac{13}{60}$ or $21.\overline{6}\%$.

Common Misconceptions or Errors

- Students may not understand how the two-way relative frequency table connects with true positives/false positives and false negatives/true negatives.
- Students may not understand how to create the proper ratio for a given purpose.
- Students may not understand how to connect the ratio to the context.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1)

A diabetes diagnostic test can sometimes lead to false negative and false positive results. A city is implementing a new diabetes diagnostic test for all its residents. Five percent of people in the city actually have the disease. The test has a 97% accuracy rating for positive results and a 7% false positive rate.

Part A. Create a two-way relative frequency table to summarize the data.

Part B. If a person tests positive, what is the probability that the person has diabetes? Explain your reasoning.

Part C. If a person tests negative, what is the probability that the person has diabetes? Explain your reasoning.

Part D. Would you recommend the city use the diabetes diagnostic? Why or why not?

Instructional Items

Instructional Item 1

In 2016, a clinical study in a small town of New Hampshire found that about 13% of its population had the flu and 87% did not. The study also showed that 19% of the people who had the flu tested negative, while 8% of the people who did not have the flu tested positive. The relative frequency table below summarizes the data.

	Positive	**Negative**	**Total**
Has the Flu	10.53%	2.47%	13%
Do Not Have the Flu	6.96%	80.04%	87%

Determine the ratio of true positives to false positive and interpret that in terms of the context.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.3.4

Benchmark

MA.912.DP.3.4 Given a relative frequency table, construct and interpret a segmented bar graph.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.3.1
- MA.912.DP.3.2
- MA.912.DP.3.3
- MA.912.DP.5.11

Terms from the K-12 Glossary

- Frequency table

Vertical Alignment

Previous Benchmarks

- MA.912.DP.1.1

Next Benchmarks

- MA.912.DP.3.5, MA.912.DP.4.5

Purpose and Instructional Strategies

In Algebra I, students selected appropriate methods to represent categorical data including bar graphs and segmented bar graphs. In Math for Data and Financial Literacy, students take data represented in a two-way frequency table and use this data to construct a segmented bar graph and interpret the data.

- Instruction includes the use of problems with relevant, real-world contexts *(MTR.7.1)*.
- **Segmented Bar Graph**
  A segmented bar graph is used to compare two categories within a data set. Each bar will consist of all the data in that category. Each bar will be partitioned to show the different percentages of each type of data in that category. Each bar will show 100% of the data in that category *(MTR.2.1)*.
  - A person takes part in a medical trial that tests the effect of a medicine on a disease. Half the people are given medicine and the other half are given a placebo, which has no effect on the disease. The medicine has a 62% chance of curing someone. But people who do not get the medicine still have a 10% chance of getting well. There are 120 people in the trial and they all have the disease. Construct a two-way relative frequency table to summarize the data. Use a tree diagram to help organize the information. The two-way relative frequency table that summarizes the data is shown below.

	Gets Better	**Doesn't Get Better**	**Total**
Gets Medicine	31%	19%	50%
Gets Placebo	5%	45%	50%
Total	36%	64%	100%

  - To construct the segmented bar graph, students can choose to have the categories as Gets Medicine and Gets Placebo or students can choose Gets Better and Doesn't Get Better as the two categories. For this example, we will choose Gets Medicine and Gets Placebo as the comparative categories.
  - To construct the segmented bar for Gets Better, students will need to determine the conditional relative frequencies for getting better with the condition that a person gets the medicine and for not getting better with the same condition. The

two percentages together, should add up to 100%

(Gets Better | Gets Medicine)= $\frac{31}{50} = 62\%$

(Doesn't Get Better | Gets Medicine)= $\frac{19}{50} = 38\%$

Students will repeat the process with the condition that a person gets the placebo.

(Gets Better | Gets Placebo)= $\frac{5}{50} = 10\%$

(Doesn't Get Better | Gets Placebo)= $\frac{45}{50} = 90\%$

- Students will need to create a key to show which segment belongs to Gets Better and which segment belongs to doesn't get better. This is typically done with different colors or markings for the segments.

Common Misconceptions or Errors

- Students may use the joint relative frequencies when segmenting the bars. In this case, the segments would not add up to 100%.
- Students may forget to make a key defining the segment pieces.
- Students may have trouble determining the conditional relative frequencies or may mix up the conditions.

Instructional Tasks

Instructional Task 1 (MTR.4.1, MTR.7.1, MTR.2.1)

A diabetes diagnostic test can sometimes lead to false negative and false positive results. A city is implementing a new diabetes diagnostic test for all its residents. Five percent of people in the city actually have the disease. The test has a 97% accuracy rating for positive results and a 7% false positive rate.

Part A. Create a two-way relative frequency table to summarize the data.

Part B. Use Part A to create a segmented bar graph representing the data.

Part C. What do each of the joint frequencies mean in terms of the context?

Part D. Would you recommend the city use the diabetes diagnostic? Why or why not?

Instructional Items

Instructional Item 1

The relative frequency table below describes risk factors for obesity associated with age.

	Risk of Obesity	**Not at Risk of Obesity**	**Total**
Ages 18 to 24	21%	29%	50%
Ages 25 to 44	31%	19%	50%
Total	52%	48%	100%

Use the relative frequency table above to create a segmented bar graph representing the data.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

MA.912.DP.5 *Determine methods of data collection and make inferences from collected data.*

MA.912.DP.5.11

Benchmark

MA.912.DP.5.11	Evaluate reports based on data from diverse media, print and digital resources by interpreting graphs and tables; evaluating data-based arguments; determining whether a valid sampling method was used; or interpreting provided statistics.

Example: A local news station changes the y-axis on a data display from 0 to 10,000 to include data only within the range 7,000 to 10,000. Depending on the purpose, this could emphasize differences in data values in a misleading way.

Benchmark Clarifications:

Clarification 1: Instruction includes determining whether or not data displays could be misleading.

Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.1.2
- MA.912.DP.2.4, MA.912.DP.2.8, MA.912.DP.2.9
- MA.912.DP.3.1, MA.912.DP.3.2, MA.912.DP.3.3, MA.912.DP.3.4

Terms from the K-12 Glossary

- Data
- Measures of center
- Measures of variability
- Population
- Random sampling

Vertical Alignment

Previous Benchmarks

- MA.912.DP.1.1

Next Benchmarks

Purpose and Instructional Strategies

In middle grades and Algebra I, students created and analyzed student gathered data or data from outside sources for various situations. In Math for Data and Financial Literacy, students continue this exploration by analyzing different media resources and determine whether data representation from the source is misleading.

- Instruction includes the use of various media resources such as newspapers, magazines, internet, etc. Students should be able to identify whether a valid sampling method was used to collect data. Students do need to identify the type of sampling method, only if it is valid or if it creates bias.
  - For example, if a sample is collected through a voluntary process, the sample is likely to contain more emotional or extreme responses. Participants who are not as passionate about the topic, are not as likely to take the time to respond. This could lead to a biased sample.
- Instruction includes the exploration of misleading graphs and why they are misleading. Some types of misleading graphs are those that use a vertical axis that does not start at 0, uses inconsistent scales, uses different shapes or images instead of bars in bar charts, etc. Students should also discuss what the impact misleading graphs and data may have on society when they are allowed to be published as fact.
- When looking at graphs that are misleading, students should have experience in deciding what can be done, if anything, to correct the presentation of the graph so that it is more accurate.

Common Misconceptions or Errors

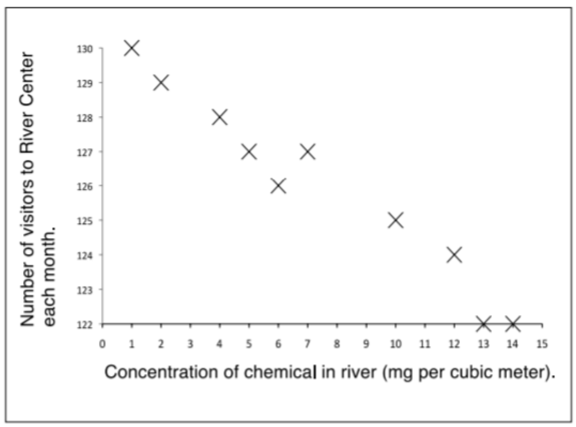
- Students may not understand that there are alternative interpretations of data and statistics and that some may be biased.
- Students may not be able to recognize survey questions that push participants towards a certain answer.

Instructional Tasks

Instructional Task 1

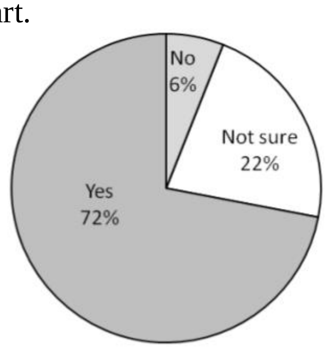
The manager of the Riverside Center is concerned about visitor numbers. He is certain the Center's popularity has been badly affected by an increase in river pollution. He feels the local environmental agency should do something about it. To support his argument, he measured the chemical concentration in the river each month. He also counted the number of people visiting the Center over several months. He used the results to draw this chart.

Resource: Interpreting Data: *Muddying the Waters 2015*, MARS, Shell Center, University of Nottingham.

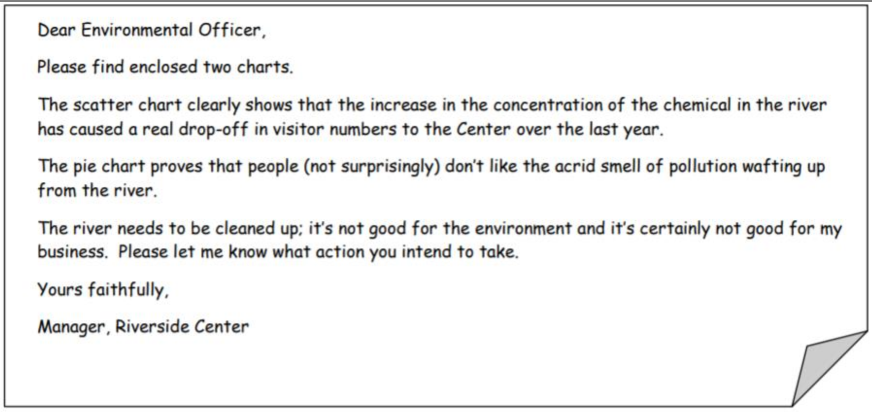


Scatter chart: Chemical concentration and number of visitors

At the same time the manager asked 18 visitors this question: 'The odor you can smell originates from the pollution in the river. Is it spoiling your enjoyment of the Center?' He displayed the results as a pie chart.



The Center Manager writes to the Environmental Officer to try to get something done about the river pollution:


Dear Environmental Officer,

Please find enclosed two charts.

The scatter chart clearly shows that the increase in the concentration of the chemical in the river has caused a real drop-off in visitor numbers to the Center over the last year.

The pie chart proves that people (not surprisingly) don't like the acrid smell of pollution wafting up from the river.

The river needs to be cleaned up; it's not good for the environment and it's certainly not good for my business. Please let me know what action you intend to take.

Yours faithfully,

Manager, Riverside Center

Part A. Describe in detail what you think the two charts show.

Part B. Do you think the Riverside Center Manager's argument is fair? Explain your reasoning.

Instructional Items

Instructional Item 1

A local newspaper editor wants to determine the proportion of its readers who favor paying for improving local schools by increasing the property tax. Which technique would likely provide the most accurate, unbiased sample?

a. The newspaper sends surveys to 1,000 subscribers at random.
b. A reporter interviews 1,000 people walking along the street near the newspaper offices.
c. Readers are asked to email the newspaper and express their opinion; 1,000 of these responses are selected at random.
d. The editor selects 1,000 phone numbers at random from the phone directory covering the area. Each number is called and the respondents are interviewed.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*