THE B.E.S.T. STANDARDS

Benchmarks for Excellent Student Thinking

Florida's B.E.S.T. Standards for 6-12 Mathematics

July 22, 2020

FLORIDA DEPARTMENT OF EDUCATION

Pop Quiz

- Using 10 words or less, describe one of the stages regarding arithmetic operations within NSO.
  - To respond, please visit menti.com and use the code 43 41 57.

Responses

- Explore
  - Concrete
  - Hands on
  - Manipulatives
  - Flexibility, CRA (Polk Math Jedis)
  - Gives students a way to dig into the math!
  - Drawings, discussions, estimations
  - No fluency expectations
- Automaticity
  - Recalling specific facts from memory

Responses

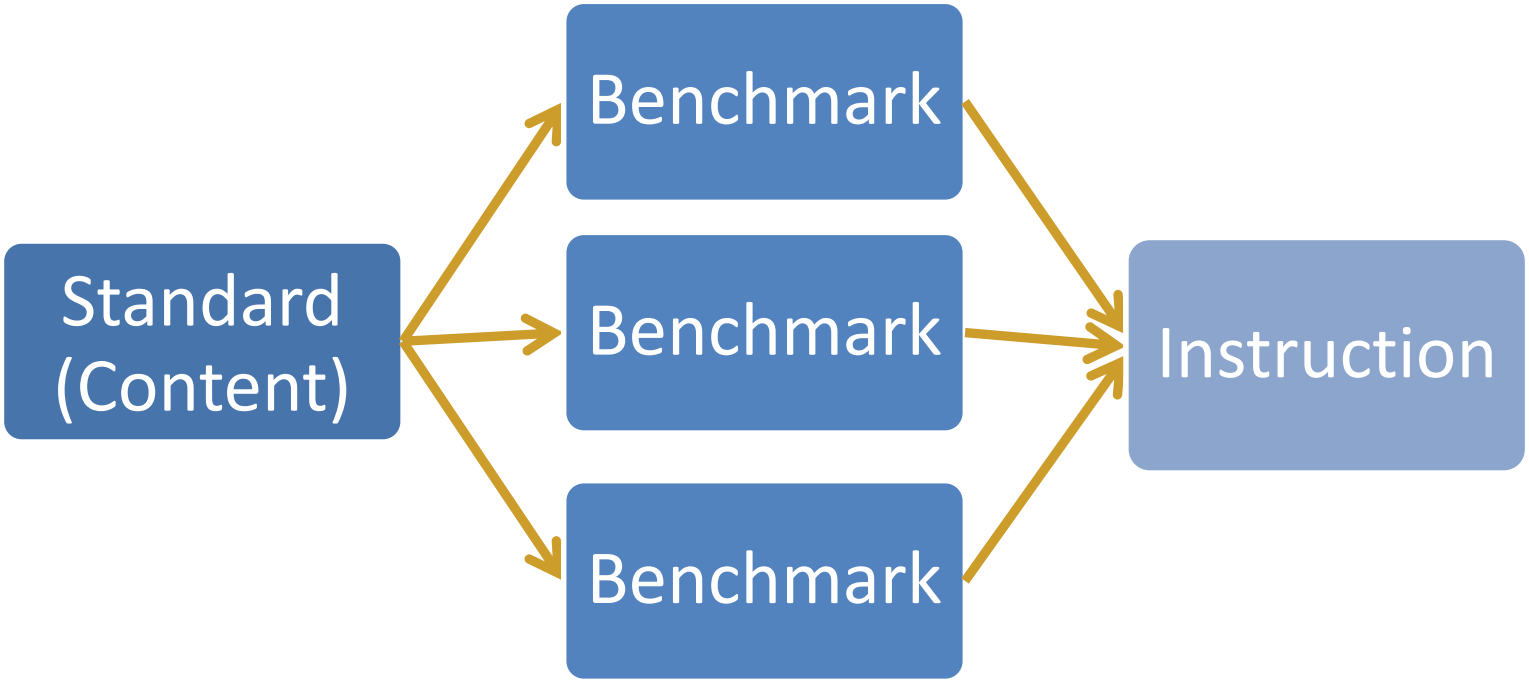
- Procedural Reliability (Escambia Math Rockettes)
  - Strategy that is accurate
  - Students becomes accurate and reliable
  - Includes models, examples, multiple methods (Madison)
  - Drawings, discussions, estimations
- Procedural Fluency
  - Choosing the most efficient method/strategy for the context (SLPS)
  - Consistently apply a method
  - Efficiency, accurate, a standard algorithm
  - How using different methods will support learning

Objectives

- Gain an understanding of the Florida's B.E.S.T. Standards for 6-12 Mathematics
  - Benchmark language, examples and clarifications
  - Horizontal and vertical progressions
  - Appendices

Current Instructional Design

B.E.S.T. Instructional Design



B.E.S.T. Standards for 6-12 Mathematics: Benchmark Language, Examples & Clarifications

Intentional Benchmark Language

Current Language	B.E.S.T. Language
• "the" standard algorithm	• "a" standard algorithm
• "shapes", "objects", "figures"	• two- and three-dimensional figures
• Dot plot	• Line plot
• "use formulas" for area, volume and surface area	• "apply formulas" for area, volume and surface area
• Single benchmark covering all functions	• Several benchmark specific to each type of function

Benchmark Examples

- Written as example questions that could be used as tasks within the classroom or ways to solve a problem that could be used to understand intention of benchmark

Benchmark Clarifications

- Written to support instruction, including the extent of which benchmark should be taught

Examples

MA.6.NSO.2.2 Extend previous understanding of multiplication and division to compute products and quotients of positive fractions by positive fractions, including mixed numbers, with procedural fluency.

Benchmark Clarifications:
Clarification 1: Instruction focuses on making connections between visual models, the relationship between multiplication and division, reciprocals and algorithms.

MA.7.AR.3.2 Apply previous understanding of ratios to solve real-world problems involving proportions.

Example: Scott is mowing lawns to earn money to buy a new gaming system and knows he needs to mow 35 lawns to earn enough money. If he can mow 4 lawns in 3 hours and 45 minutes, how long will it take him to mow 35 lawns? Assume that he can mow each lawn in the same amount of time.

Example: Ashley normally runs 10-kilometer races which is about 6.2 miles. She wants to start training for a half-marathon which is 13.1 miles. How many kilometers will she run in the half-marathon? How does that compare to her normal 10K race distance?

Examples

MA.8.DP.2.3 Solve real-world problems involving probabilities related to single or repeated experiments, including making predictions based on theoretical probability.

Example: If Gabriella rolls a fair die 300 times, she can predict that she will roll a 3 approximately 50 times since the theoretical probability is $\frac{1}{6}$.

Example: Sandra performs an experiment where she flips a coin three times. She finds the theoretical probability of landing on exactly one head as $\frac{3}{8}$. If she performs this experiment 50 times (for a total of 150 flips), predict the number of repetitions of the experiment that will result in exactly one of the three flips landing on heads.

Benchmark Clarifications:

Clarification 1: Instruction includes making connections to proportional relationships and representing probability as a fraction, percentage or decimal.

Clarification 2: Experiments to be repeated are limited to tossing a fair coin, rolling a fair die, picking a card randomly from a deck with replacement, picking marbles randomly from a bag with replacement and spinning a fair spinner.

Clarification 3: Repetition of experiments is limited to two times except for tossing a coin.

Examples

MA.912.NSO.1.1 Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.

Benchmark Clarifications:
Clarification 1: Instruction includes the use of technology when appropriate.
Clarification 2: Refer to the K-12 Formulas (Appendix E) for the Laws of Exponents.
Clarification 3: Instruction includes converting between expressions involving rational exponents and expressions involving radicals.
Clarification 4: Within the Mathematics for Data and Financial Literacy course, it is not the expectation to generate equivalent numerical expressions.

MA.912.DP.5.11 Evaluate reports based on data from diverse media, print and digital resources by interpreting graphs and tables; evaluating data-based arguments; determining whether a valid sampling method was used; or interpreting provided statistics.

Example: A local news station changes the *y*-axis on a data display from 0 to 10,000 to include data only within the range 7,000 to 10,000. Depending on the purpose, this could emphasize differences in data values in a misleading way.

Benchmark Clarifications:
Clarification 1: Instruction includes determining whether or not data displays could be misleading.

Examples

MA.912.AR.1.2 Rearrange equations or formulas to isolate a quantity of interest.

Algebra 1 Example: The Ideal Gas Law $PV = nRT$ can be rearranged as $T = \frac{PV}{nR}$ to isolate temperature as the quantity of interest.

Example: Given the Compound Interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, solve for P.

Mathematics for Data and Financial Literacy Honors Example: Given the Compound Interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, solve for t.

Benchmark Clarifications:

Clarification 1: Instruction includes using formulas for temperature, perimeter, area and volume; using equations for linear (standard, slope-intercept and point-slope forms) and quadratic (standard, factored and vertex forms) functions.

Clarification 2: Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

Examples

MA.912.FL.2.5 Develop budgets that fit within various incomes using spreadsheets and other technology.

Example: Develop a budget spreadsheet for your business that includes typical expenses such as rental space, transportation, utilities, inventory, payroll, and miscellaneous expenses. Add categories for savings toward your own financial goals, and determine the monthly income needed, before taxes, to meet the requirements of your budget.

Benchmark Clarifications:
Clarification 1: Instruction includes budgets for a business and for an individual.
Clarification 2: Instruction includes taking into account various cash management strategies, such as checking and savings accounts, and how inflation may affect these strategies.

MA.912.GR.4.5 Solve mathematical and real-world problems involving the volume of three-dimensional figures limited to cylinders, pyramids, prisms, cones and spheres.

Example: A cylindrical swimming pool is filled with water and has a diameter of 10 feet and height of 4 feet. If water weighs 62.4 pounds per cubic foot, what is the total weight of the water in a full tank to the nearest pound?

Benchmark Clarifications:
Clarification 1: Instruction includes concepts of density based on volume.
Clarification 2: Instruction includes using Cavalieri's Principle to give informal arguments about the formulas for the volumes of right and non-right cylinders, pyramids, prisms and cones.

Examples

MA.912.F.1.2 Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.

Algebra 1 Example: The function $f(x) = \frac{x}{7} - 8$ models Alicia's position in miles relative to a water stand x minutes into a marathon. Evaluate and interpret for a quarter of an hour into the race.

Benchmark Clarifications:
Clarification 1: Problems include simple functions in two-variables, such as $f(x, y) = 3x - 2y$.
Clarification 2: Within the Algebra 1 course, functions are limited to one-variable such as $f(x) = 3x$.

MA.912.LT.5.4 Perform the set operations of taking the complement of a set and the union, intersection, difference and product of two sets.

Benchmark Clarifications:
Clarification 1: Instruction includes the connection to probability and the words AND, OR and NOT.

MA.912.C.1.2 Determine the value of a limit if it exists algebraically using limits of sums, differences, products, quotients and compositions of continuous functions.

Example: Find $\lim_{x \to \pi} (\sin x \cos x + \tan x)$.

Discussion

- How will understanding the benchmark clarifications and examples help with successful implementation of the B.E.S.T. Standards for Mathematics?
  - To respond, please visit menti.com and use the code 28 81 67.

Responses

- Intentional and focused instruction
- Clarity on instruction expectations (SLPS)
- Common language and understanding
- Consistency
- Limitations of the benchmarks
- Clear instruction, alignment and progression
- Clarify the depth of each benchmark
- Help identify appropriate levels of rigor
- Clear expectations for what is to be taught and learned (Gadsden)

Responses

- Helps avoid the over/under teaching of the benchmark
- Understand the scope and intent of the benchmark
- Allows for better planning to ensure teaching understanding leading to successful implementation
- Clear instruction, alignment and progression
- Teacher will know what to teach instead of looking at test specs; everything all in one place
- Decrease misconceptions and provide expectations
- Help reinforce precise teaching strategies and ensuring that we are not teaching “unnecessary content”
- Help support teachers in their instruction and understanding

Horizontal & Vertical Progressions

Horizontal Progression

- Each grade level or course indicates the areas of emphasis
- Intentional progression of content within the strands and across the strands

In grade 7, instructional time will emphasize five areas:

(1) recognizing that fractions, decimals and percentages are different representations of rational numbers and performing all four operations with rational numbers with procedural fluency;
(2) creating equivalent expressions and solving equations and inequalities;
(3) developing understanding of and applying proportional relationships in two variables;
(4) extending analysis of two- and three-dimensional figures to include circles and cylinders and
(5) representing and comparing categorical and numerical data and developing understanding of probability.

Horizontal Progression – Example

MA.7.AR.4 Analyze and represent two-variable proportional relationships.				
MA.7.AR.4.1	MA.7.AR.4.2	MA.7.AR.4.3	MA.7.AR.4.4	MA.7.AR.4.5
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.	Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.	Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.	Given any representation of a proportional relationship, translate the representation to a written description, table or equation.	Solve real-world problems involving proportional relationships.

Horizontal Progression – Example

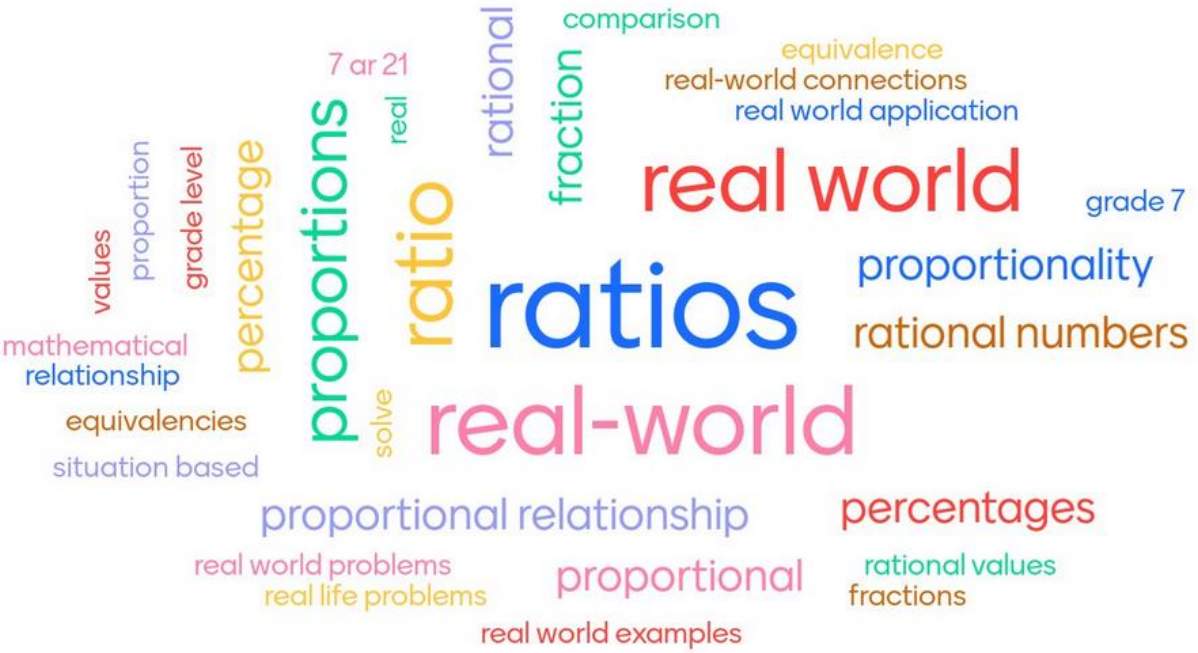
MA.7.NSO.1.2		MA.7.AR.3.1		MA.7.GR.1.5		MA.7.DP.1.3		MA.7.DP.2.3
Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.	↔	Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.	↔	Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.	↔	Given categorical data from a random sample, use proportional relationships to make predictions about a population.	↔	Find the theoretical probability of an event related to a simple experiment.

Discussion

- In thinking about the prior example, what is one similarity within the benchmarks?
  - To respond, please visit menti.com and use the code 47 24 28.

Responses

What is one similarity within the benchmarks?



Additional Discussion

- In thinking about the prior example,
  - what are some similarities within the benchmarks?
  - what are some differences within the benchmarks?
  - what other benchmarks, within the grade level, would you consider part of the horizontal progression?
  - how will knowing the horizontal progression help to increase student achievement for all students?

Vertical Progression

- Intentional progression of content from one year to the next or across multiple grade levels
  - Progression within the same strand and across strands
- Intentional progression of content among the strands from one grade level to the next

Vertical Progression – Example

MA.K.AR.2	→	MA.1.AR.2	→	MA.2.AR.2	→	MA.3.AR.2	→	MA.4.AR.2
Develop an understanding of the equal sign.		Develop an understanding of the relationship between addition and subtraction.		Demonstrate an understanding of equality and addition and subtraction.		Develop an understanding of equality and multiplication and division.		Demonstrate an understanding of equality and operations with whole numbers.

MA.5.AR.2	→	MA.6.AR.1	→	MA.7.AR.1	→	MA.8.AR.1	→	MA.912.AR.1
Demonstrate an understanding of equality, the order of operations and equivalent numerical expressions.		Apply previous understanding of arithmetic expressions to algebraic expressions.		Rewrite algebraic expressions in equivalent forms.		Generate equivalent algebraic expressions.		Interpret and rewrite algebraic expressions and equations in equivalent forms.

Vertical Progression – Example

MA.K.GR.1	→	MA.1.GR.1	→	MA.2.GR.1	→	MA.3.GR.1	→	MA.4.GR.1
Identify, compare and compose two- and three-dimensional figures.		Identify and analyze two- and three-dimensional figures based on their defining attributes.		Identify and analyze two-dimensional figures and identify lines of symmetry.		Describe and identify relationships between lines and classify quadrilaterals.		Draw, classify and measure angles.

MA.5.GR.1	→	MA.6.GR.1	→	MA.7.GR.1	→	MA.8.GR.1	→	MA.912.GR.1
Classify two-dimensional figures and three-dimensional figures based on defining attributes.		Apply previous understanding of the coordinate plane to solve problems.		Solve problems involving two-dimensional figures, including circles.		Develop an understanding of the Pythagorean Theorem and angle relationships involving triangles.		Prove and apply geometric theorems to solve problems.

Discussion

- In thinking about vertical progression, why might it be important to understand the intentional placement of standards from Kindergarten to grade 12?
  - To respond, please type into the question box.

Responses

- Teachers need to understand the knowledge students should walk in the door with and should know how their instruction impacts the student in following years
- Consistency
- So that we know what grade level is responsible to what piece of the standard
- Building on the foundation to build on the depth of knowledge and complexity
- So that teachers know where it came from and where it is going to and teach materials in a consistent manner (SLPS)
- Allows for teachers to understand what instruction was before, where the learning is going and the depth of instruction that is needed
- Strategic MTSS, interventions, gaps in learning, and planning
- Differentiated Instruction

Responses

- It's important for each grade level to understand where students have been and where they're going. This knowledge helps teachers to differentiate instruction so that all students have opportunity for success with the benchmarks
- Teachers will need to know what content knowledge the students are bringing to the grade level and what they need to be successful at the next grade level. The content knowledge is being carefully grown
- To see the progression from where we have been to where we are going
- Depth of understanding and level difficulty. Teachers can see the importance of their role in developing the students understanding of the benchmarks
- The intentional placement of the standards would aide with understanding exactly what the prerequisites for the skills are. In addition, it helps ensure that we are better able to meet the needs of all students given any academic gap.

Vertical Progression – Example

MA.8.NSO.1.2	MA.8.AR.2.3	MA.8.GR.1.2	MA.912.NSO.1.1	MA.912.AR.3.1
Plot, order and compare rational and irrational numbers, represented in various forms.	Given an equation in the form of $x^2 = p$ and $x^3 = q$, where p is a whole number and q is an integer, determine the real solutions.	Apply the Pythagorean Theorem to solve mathematical and real-world problems involving the distance between two points in a coordinate plane.	Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.	Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

Discussion

- What is at least one benchmark prior to grade 8 (including K-5) you would include in this vertical progression?
  - To respond, please visit menti.com and use the code 68 59 95.

Responses

- Plotting points on the coordinate plane
  - MA.5.GR.4
- Plotting points on the number line
  - MA.4.NSO.1.3
  - MA.4.NSO.1.5
  - MA.4.FR.1.4
  - MA.5.NSO.1.4
  - MA.6.NSO.1.1
- One- and two-step equations
- Understanding of exponents
  - MA.7.NSO.1.1

Responses

- MA.4.NSO.1.2 (SLPS)
- Classifying numbers
- Measuring distance between two points on the coordinate plane
- MA.K.DP.1.1
- MA.4.GR.2.1
- MA.2.NSO.1
- MA.7.NSO.2.1
- MA.7.NSO.1.2

Additional Discussion

- In thinking about the prior example,
  - what are some similarities within the benchmarks?
  - what are some differences within the benchmarks?
  - what other benchmarks would you consider part of the vertical progression?
  - how will knowing the vertical progression help to increase student achievement for all students?

Progressions from Secondary to Post-Secondary

- Florida Student Success Center established workgroups to identify current challenges in mathematics pathways and develop policy and practice recommendations to improve student achievement across Florida's education systems.
- Within 9-12 benchmarks were developed to support the work of the Mathematics Re-Design project

https://www.floridacollegesystemfoundation.org/fssc-math-redesign

Progressions from Secondary to Post-Secondary

- Three pathways at post-secondary level
  - College Algebra
    - Creating connections of equations and functions to real-world context and modeling
  - College Statistics
    - Strengthening basic algebraic skills as relates to data analysis; developing understanding of probability theory; analyzing and interpreting statistical graphs and tables
  - College Liberal Arts
    - Creating connections within various strands including algebraic reasoning, geometric reasoning, data analysis & probability and logic and set theory

Activity

- In thinking about the three pathway courses, find at least 3 benchmarks that support one of these courses. To respond, please type into the question box.
  - College Algebra
  - College Statistics
  - College Liberal Arts

Responses

- College Algebra- MA.912.AR.9.1 Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically
- MA.912.AR.1.1 9-12 Algebraic Reasoning Standards
- MA.912.LT.1.1 Apply recursive and iterative thinking to solve problems. & MA.912.DP.5.7 Compare and contrast surveys, experiments and observational studies
- College Algebra - MA.3.AR.1.2
- College Algebra - MA.912.AR.1.2; MA.912.AR.1.3; MA.912.AR.1.4
- College Statistics: MA.5.DP.1.2
- College Statistics – MA.912.DP.3.1; MA.912.DP.3.2; MA.912.DP.3.3

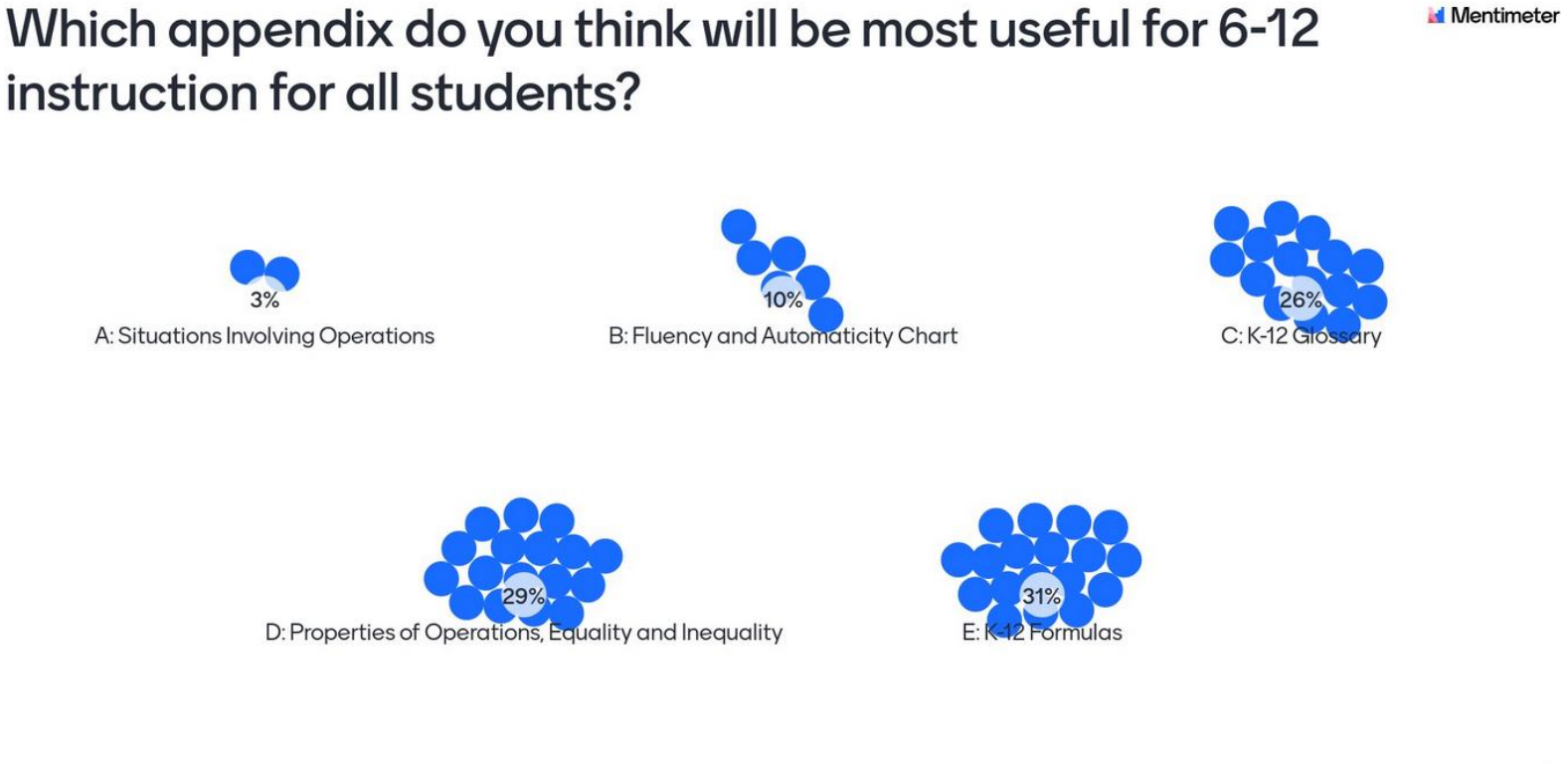
Florida's B.E.S.T. Standards for 6-12 Mathematics: Appendices

Discussion

- Which appendix do you think will be most useful for 6-12 instruction for all students?
  - To respond, please visit menti.com and use the code 87 96 48.

A. Situations Involving Operations
B. Fluency and Automaticity Chart
C. K-12 Glossary
D. Properties of Operations, Equality and Inequality
E. K-12 Formulas

Responses


Which appendix do you think will be most useful for 6-12 instruction for all students?
Mentimeter
3%
A: Situations Involving Operations
10%
B: Fluency and Automaticity Chart
26%
C: K-12 Glossary
29%
D: Properties of Operations, Equality and Inequality
31%
E: K-12 Formulas
31

Additional Activity

- Using your copy of Florida's B.E.S.T. Standards for Mathematics, find one thing in each appendix you feel will be useful for 6-12 instruction for all students. How does this differ from what you chose for K-5?
  - A. Situations Involving Operations
  - B. Fluency and Automaticity Chart
  - C. K-12 Glossary
  - D. Properties of Operations, Equality and Inequality
  - E. K-12 Formulas

Appendix A: Situations Involving Operations

- Add to
- Take from
- Put together
- Compare
- Equal groups
- Arrays
- Multiplicative comparisons

Appendix B: Fluency and Automaticity Chart

6	Plot, order and compare rational numbers	*Procedural Fluency:* Positive multi-digit decimals, including using a standard algorithm *Procedural Fluency:* Positive fractions, including mixed numbers and fractions greater than 1 *Procedural Fluency:* Integers	
7		*Procedural Fluency:* Rational numbers	
8	Plot, order and compare rational and irrational numbers	*Procedural Fluency:* Numbers expressed in scientific notation *Procedural Fluency:* Laws of Exponents	

Appendix C: K-12 Glossary

Vocabulary	Definition	Example
polynomials	The sum or difference of terms which have variables raised to non-negative integer powers and which have coefficients that may be real or complex.	$5x^3 - 2x^2 + x - 13$ $x^2y^3 + xy$ $(1 + i)a^2 + ib^2$
population (in data analysis)	The entire set of cases or individuals under consideration in a statistical analysis.	A poll given to a sample of voters is designed to measure the preferences of the population of all voters.
prime factorization	The expression of a number as the product of prime factors.	The prime factorization of 72 is $2 \times 2 \times 2 \times 3 \times 3$.
prime number	A whole number greater than 1 that is not divisible by any whole number other than 1 and itself.	17 is a prime number. 16 is not a prime number.
principal square roots	The principal square root is the positive square root of a positive real number.	

Appendix D: Properties of Operations, Equality and Inequality

Property of Inequality	**Example**
Asymmetric property of inequality	If $a > b$, then $b < a$.
Transitive property of inequality	If $a > b$ and $b > c$, then $a > c$.
Addition property of inequality	If $a > b$, then $a + c > b + c$.
Subtraction property of inequality	If $a > b$, then $a - c > b - c$.
Multiplication property of inequality	If $a > b$ and $c > 0$, then $a \times c > b \times c$. If $a > b$ and $c < 0$, then $a \times c < b \times c$.
Division property of inequality	If $a > b$ and $c > 0$, then $a \div c > b \div c$. If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Appendix E: K-12 Formulas

- Area of two-dimensional figures
- Surface area and volume for three-dimensional figures
- Laws of Exponents

Appendix E: K-12 Formulas

Laws of Exponents (where m and n are integers and a and b are nonzero real numbers)	
Product of powers	$a^m \cdot a^n = a^{m+n}$ and conversely $a^{m+n} = a^m \cdot a^n$
Quotient of powers	$\frac{a^m}{a^n} = a^{m-n}$ and conversely $a^{m-n} = \frac{a^m}{a^n}$
Power of a power	$(a^m)^n = a^{m \cdot n}$ and conversely $a^{m \cdot n} = (a^m)^n$
Power of a product	$(ab)^m = a^m \cdot b^m$ and conversely $a^m \cdot b^m = (ab)^m$
Power of a quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ and conversely $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
Negative exponent	$a^{-1} = \frac{1}{a}$ and conversely $\frac{1}{a} = a^{-1}$ $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$ and conversely $\frac{b}{a} = \left(\frac{a}{b}\right)^{-1}$
Identity exponent	$a^1 = a$
Zero exponent	$a^0 = 1$
Rational, Fractional exponent	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ and conversely $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$, where $a > 0$ $a^{\frac{m}{n}} = \sqrt[n]{(a^m)}$ and conversely $\sqrt[n]{(a^m)} = a^{\frac{m}{n}}$, where $a > 0$

Questions?

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